KASU JOURNAL OF MATHEMATICAL SCIENCES (KJMS) VOL. 6, ISSUE 2, December 2025
ISSN 2734-3839 (Print), ISSN 2735-962X (E-Copy)
http://www.journal.kasu.edu.ng/index.php/kjms DOI/10.5281/zenodo.12627471.svg

KASU JOURNAL OF MATHEMATICAL SCIENCES

Inventory Ordering Policies of Delayed Deteriorating Items with Ramp Type Demand, Price Discount and Complete Backlogging

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Abstract
This study presents the inventory ordering policies of delayed/non- instantaneous deteriorating items with Ramp Type demand, Price discount and complete backlogging. The premise of a price discount on the selling price for the proposed model acts as a motivator for both wholesalers and retailers, which in turn contributes to the notable increase in demand for the items put up for sale. The Ramp Type demand rate is very common when a brand of consumer goods is introduced to the market and then their demand increases linearly at first, before the market stabilizes and the demand becomes constant until the end of the inventory cycle. A suitable iterative procedure is used to solve the mathematical solutions in order to produce numerical examples on the applications of the model. Sensitivity analysis would be conducted to highlight the impact of changes in the model's solutions caused by minor changes in the system parameters. The suggested model is developed incorporating both pre and post deteriorating as well as discount on unit selling price under complete backlog, with analytical solutions provided.

Keywords: Delayed deteriorating, Ramp Type demand, Price discount, complete backlogging

1. Introduction

All businesses maintain inventory, which comprises their raw materials, ongoing projects, operational supplies, and completed commodities. Materials or products utilized by a business for manufacturing and selling are referred to as inventory. Inventory models are vital in factories, markets, businesses, and industries. One of the key components of inventory control is product perishability. A variety of effects on stock, including damage, spoiling, decay, declining usefulness, and many more, can be attributed to degradation in general. Grain, fruits, electronics, and radioactive materials all gradually lose their potential as time goes on. (Vandana 2018).
A ramp-type demand rate occurs when a new brand of consumer products enters the market. It rises at the start of the season for a predetermined amount of time, say, and then stays the same for the remainder of the season. The Ramp-type demand rate has been used to products like new consumer goods brands, etc., whose demand spikes for a short while before falling. This idea was the subject of much inquiry. Every business that deals with inventory has a number of basic issues. One of them is the choice of price. When to review the pricing as the season progresses and how much to charge per unit are decided. It constantly aims for a marginal price—not too high to turn off potential customers, nor too low to lose out on possible earnings. Price can therefore be regarded as a crucial instrument for influencing demand. (Panda et al. 2009)

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The majority of academics have believed that there is a total backlog of shortages. Sachan later permitted shortages in the EOQ model. In actuality, some clients are unwilling to wait during the scarcity period. However, there are a lot of instances where clients are ready to wait for the backlog of goods. Thus, in the EOQ model, Abad first took the impatience of the consumer into account. Chang and Dye created an inventory model where the reciprocal of the linear function of waiting time represents the proportion of customers who would be willing to accept backlog. (Panda et al. 2009)

2. Literature Review

The issue of decaying inventory was initially explored by Ghare and Shrader (1963), who created an EOQ model with a constant decay rate. Covert and Philip (1973) expanded upon this model by incorporating a two-parameter Weibull distribution. Later, Tadikmalla (1978) further generalized the model using a three-parameter gamma distribution to represent deterioration time. Hollier and Mark (1983) introduced an inventory replenishment model for deteriorating items in a declining market, while Cheng and Chen (2004) examined deteriorating items in a periodic review setting with shortages. In the classical EOQ model, Harris (1915) assumed a constant demand rate, but Donaldson (1977) extended it to accommodate a linear trend in demand. Hill (1995) later developed the Ramp-type demand model. Other significant contributions include the models of Mandal and Pal (1998), Wuu Wu et al. (1999), Chaudhri et al. (2006), and Skouri et al. (2009).

The EOQ model was originally based on a constant demand rate. However, in reality, demand for physical goods may depend on stock levels, time, price, or a combination of these factors. Levin et al. (1972) noted that the presence of inventory often motivates people, suggesting that large displays in supermarkets can encourage consumers to buy more. This was confirmed by Silver and Peterson (1985), who found that retail sales tend to correlate with the amount of inventory displayed.

Numerous researchers have explored the impact of pricing strategies on inventory models, including Arcelus and Srinivasan (1998), Shah and Shah (1993), and Wee and Law (2001). Khouja (2000) studied an inventory problem where multiple discounts are used to sell excess inventory. Literature has also examined the independent effects of temporary price discounts on inventory policies under various assumptions (Weatherford and Bodily, 1992; Petruzzi and Dada, 1999). Neff (2000) suggested that discounts can boost sales by increasing demand, which accelerates inventory depletion and reduces holding costs.

Many studies have assumed that shortages are fully backlogged, but Sachan (1984) relaxed this assumption in the EOQ model. During shortage periods, some customers may be unwilling to wait, while others may accept backlogging. Abad (1996) was the first to consider customer impatience in the EOQ model. Later, Chang and Dye (1999) developed an inventory model in which the proportion of customers willing to accept backlogging is inversely related to the waiting time.

Notations

d_1 Demand rate (unit per unit time) before deterioration sets in.
d_2 Demand rate (unit per unit time) after deterioration sets in.
θ Rate of deterioration.
I_0 Initial Inventory.
r_1 Discount offer per unit before deterioration.
r_2 Discount offer per unit when deterioration is set in.
T Inventory cycle length (per unit time).
$I(t)$ Inventory level at time $[0, T_1]$

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$I_d(t)$	Inventory level at time $[T_{1,}T]$
T_1	Time deterioration sets in.
Hc	Holding cost per unit.
C	Unit cost of an item.
C_0	Cost of a unit deteriorating item.
S	Unit selling price of an item.
A_0	Ordering cost.
S_1	Unit selling price of deteriorating item
Bc	Back order cost per unit backorder per unit time.
μ	Changing point from linear demand to the constant demand

Assumptions

1. Replenishment is instantaneous.
2. Shortages are allowed.
3. The demand d_2 is assumed to be ramp type function of t.

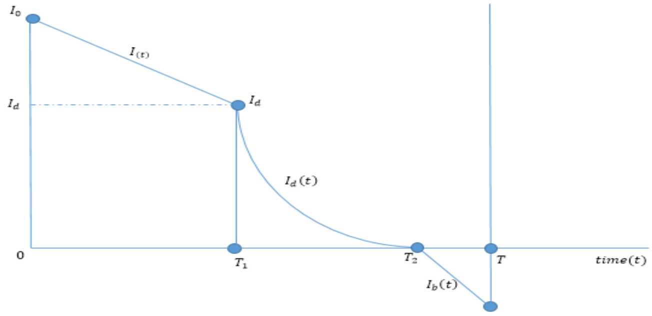
$$\text{Where, } d_2 = \begin{cases} D_0 t & t < \mu \\ D_0 \mu & t \geq \mu \end{cases} \qquad \text{(Deng et al. 2007)}$$

Where μ is the changing point from linear demand to the constant demand.

4. During $0 \leq t \leq T_1$ discount of r_1 and effect on the discounted selling price on the demand offer before deterioration set in is $\alpha_1 = (1 - r_1)^{-n_1}$, where$n_1 \in R$.
5. During $T_1 \leq t \leq T$ discount of r_2 and effect on the discounted selling price on the demand offer, when deterioration begins is $\alpha_2 = (1 - r_2)^{-n_2}$, where $n_1 \in R$. α_2 Is determine from prior knowledge of the seller, such that the demand rate is influenced with reduction rate of selling price. It is obvious that when $r_2 \to 0, \alpha_2 \to 1$, that is the demand of decreased quality items remain the same.

Diagrammatic Representation

Figure: 1



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$$\frac{dI(t)}{dt} = -d_1\alpha_1, \qquad 0 \le t \le T_1 \tag{1}$$

$$\frac{dI_d(t)}{dt} + \theta I_d(t) = -d_2\alpha_{2,} \qquad T_1 \le t \le T_2 \tag{2}$$

$$\frac{dI_b(t)}{dt} = -D_0\,\mu \qquad T_2 \le t \le T \tag{3}$$

Where, $d_2 = \begin{cases} D_0 t & t < \mu \\ D_0\,\mu & t \ge \mu \end{cases}$

With the boundary conditions, $t = 0,\ I(t) = I_{0,}\ \ t = T_1,\ I_d(t) = I_{d,}$

$t = T_2,\ I_d(t) = 0,\ t = T_2,\ I_b(t) = 0$

3. Solutions of the Equations

$$\frac{dI(t)}{dt} = -d_1\alpha_1, \quad 0 \le t \le T_1$$

$$(t) = -d_1\alpha_1 t + I_d + d_1\alpha_1 T_1$$

$$I(t) = -d_1\alpha_1(t - T_1) + I_d \tag{4}$$

$$\frac{dI_d(t)}{dt} + \theta I_d(t) = -d_2\alpha_2 \qquad T_1 \le t \le T_2$$

For $t < \mu,\ d_2 = D_0 t$

$$I_d(t) = -D_0\alpha_2\left[\frac{t}{\theta} - \frac{1}{\theta^2} + \frac{1}{\theta^2}e^{\theta(T_1-t)} - \frac{T_1 e^{\theta(T_1-t)}}{\theta}\right] + I_d\,e^{\theta(T_1-t)} \tag{5}$$

For $t \ge \mu,\ d_2 = D_0\mu$

$$I_d(t) = -D_0\alpha_2\frac{\mu}{\theta} + D_0\alpha_2\frac{\mu e^{\theta(T_2-t)}}{\theta}$$

$$I_d(t) = D_0\alpha_2\frac{\mu}{\theta}\left[e^{\theta(T_2-t)} - 1\right] \tag{6}$$

$$\frac{dI_b(t)}{dt} = -D_0\,\mu \qquad T_2 \le t \le T$$

$$I_b(t) = -D_0\mu\,t + D_0\mu\,T_2$$

$$I_b(t) = D_0\,\mu(T_2 - t) \tag{7}$$

ASSOCIATED INVENTORY COSTS

1. Ordering cost is represented as A_0

2. Holding cost, $HC = h\left[\frac{d_1\alpha_1 {T_1}^2}{2} + T_1 I_d - D_0\alpha_2\left[-\frac{\mu^2}{2\theta} - \frac{{T_1}^2}{2\theta} + \frac{1}{\theta^3} + \frac{\mu T_2}{\theta} - \frac{e^{\theta(T_1-\mu)}}{\theta^2}\left(\frac{1}{\theta} - T_1\right) - \frac{\mu e^{\theta(T_2-\mu)}}{\theta^2}\right] + \frac{I_d}{\theta} - \frac{I_d\,e^{\theta(T_1-\mu)}}{\theta} + \left(D_0\,\mu\left(T_2 T - \frac{T^2}{2} - \frac{{T_2}^2}{2}\right)\right)\right]$

3. Purchasing Cost

$PC = I_0 \times C = (I_d + d_1\alpha_1 T_1)C$

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4. Backorder cost

BC$= -S_1\int_{T_2}^{T} I_b(t)dt = -S_1\int_{T_2}^{T} D_0\mu^*(T_2-t)dt = -S_1 D_0\,\mu\left(T_2T-\frac{T^2}{2}-\frac{{T_2}^2}{2}\right)$

5. Total Cost of deteriorated items (TCD)

$$\boldsymbol{TCD = C_0\left[I_0 - d_1\alpha_1T_1 - \frac{D_0}{2}\left(2\mu T_2 - T_1^2-\mu^2\right)\right]}$$

Total Selling Revenue, SR

$$SR = S\left[\alpha_1(1-r_1)\int_0^{T_1} d_1\,dt + \alpha_2(1-r_2)\left[\int_{T_1}^{\mu} D_0 t\,dt + \int_{\mu}^{T_2} D_0\mu\,dt\right] + \alpha_2(1-r_2)\int_{T_2}^{T} D_0\mu\,dt\right]$$

$$= S\left[d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2)\left[\frac{\mu^2D_0}{2} - \frac{T_1^2D_0}{2} + D_0\mu T_2 - D_0\mu^2 + D_0\mu T - D_0\mu T_2\right]\ \right]$$

$$SR = S\left[d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2)\left[-\frac{\mu^2D_0}{2} - \frac{T_1^2D_0}{2} + D_0\mu T - D_0\mu^2\right]\ \right]$$

PROFIT FUNCTION (PF)

$PF = \frac{1}{T}(SR - PC - HC - BC - TCD - A_0)$ (Panda et al. 2009).

$PF = \frac{1}{T}\,(Total\ \text{Sales Revenue} - \text{all the inventory cost})$

$$PF = \frac{1}{T}\Bigg(S\left[d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2)\left[-\frac{\mu^2D_0}{2} - \frac{T_1^2D_0}{2} + D_0\mu T - D_0\mu^2\right]\right] - (I_d + d_1\alpha_1T_1)C - h\left[\frac{d_1\alpha_1{T_1}^2}{2} + T_1I_d - D_0\alpha_2\left[-\frac{\mu^2}{2\theta} - \frac{{T_1}^2}{2\theta} + \frac{1}{\theta^3} + \frac{\mu T_2}{\theta} - \frac{e^{\theta(T_1-\mu)}}{\theta^2}\left(\frac{1}{\theta}-T_1\right) - \frac{\mu e^{\theta(T_2-\mu)}}{\theta^2}\right] + \frac{I_d}{\theta} - \frac{I_d\,e^{\theta(T_1-\mu)}}{\theta} + \left(D_0\mu\left(T_2T - \frac{T^2}{2} - \frac{{T_2}^2}{2}\right)\right)\right] + S_1D_0\mu\left(T_2T - \frac{T^2}{2} - \frac{{T_2}^2}{2}\right) - C_0\left[I_d - \frac{D_0}{2}(2\mu T_2 - T_1^2 - \mu^2)\right] - A_0\Bigg) \quad (8)$$

NECESSARY CONDITION

To find necessary condition we differentiate PF with respect to T, then equate the result to zero to have:

$$S\left[d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2)\left[-\frac{\mu^2D_0}{2} - \frac{T_1^2D_0}{2} - D_0\mu T_2\right]\right] - (I_d + d_1\alpha_1T_1)C - h\left[\frac{d_1\alpha_1{T_1}^2}{2} + T_1I_d - D_0\alpha_2\left[-\frac{\mu^2}{2\theta} - \frac{{T_1}^2}{2\theta} + \frac{1}{\theta^3} + \frac{\mu T_2}{\theta} - \frac{e^{\theta(T_1-\mu)}}{\theta^2}\left(\frac{1}{\theta}-T_1\right) - \frac{\mu e^{\theta(T_2-\mu)}}{\theta^2}\right] + \frac{I_d}{\theta} - \frac{I_d\,e^{\theta(T_1-\mu)}}{\theta} + hD_0\mu\left(\frac{T^2}{2} - \frac{{T_2}^2}{2}\right)\right] + S_1hD_0\mu\left(\frac{T^2}{2} - \frac{{T_2}^2}{2}\right) - C_0\left[I_d - \frac{D_0}{2}(2\mu T_2 - T_1^2 - \mu^2)\right] - A_0 = 0 \quad (9)$$

$$T^2 = \frac{2}{hD_0\mu[h-S_1]}\Bigg\{S\left[d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2)\left[-\frac{\mu^2D_0}{2} - \frac{T_1^2D_0}{2} - D_0\mu T_2\right]\right] - (I_d + d_1\alpha_1T_1)C - h\left[\frac{d_1\alpha_1{T_1}^2}{2} + T_1I_d - D_0\alpha_2\left[-\frac{\mu^2}{2\theta} - \frac{{T_1}^2}{2\theta} + \frac{1}{\theta^3} + \frac{\mu T_2}{\theta} - \frac{e^{\theta(T_1-\mu)}}{\theta^2}\left(\frac{1}{\theta}-T_1\right) - \frac{\mu e^{\theta(T_2-\mu)}}{\theta^2}\right] + \frac{I_d}{\theta} - \frac{I_d\,e^{\theta(T_1-\mu)}}{\theta}\right] - C_0\left[I_d - \frac{D_0}{2}(2\mu T_2 - T_1^2 - \mu^2)\right] - A_0\Bigg\} + {T_2}^2$$

$$T^* = \sqrt{\frac{2}{hD_0\mu[h-S_1]}\{E\} + {T_2}^2} \quad (10)$$

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Where $E = \Bigg\{S\left[d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2)\left[-\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} - D_0\mu T_2\right]\right] - (I_d + d_1\alpha_1 T_1)C$

$$- h\left[\frac{d_1\alpha_1 {T_1}^2}{2} + T_1 I_d\right.$$

$$\left. - D_0\alpha_2\left[-\frac{\mu^2}{2\theta} - \frac{{T_1}^2}{2\theta} + \frac{1}{\theta^3} + \frac{\mu T_2}{\theta} - \frac{e^{\theta(T_1-\mu)}}{\theta^2}\left(\frac{1}{\theta} - T_1\right) - \frac{\mu e^{\theta(T_2-\mu)}}{\theta^2}\right] + \frac{I_d}{\theta} - \frac{I_d\, e^{\theta(T_1-\mu)}}{\theta}\right]$$

$$- C_0\left[I_d - \frac{D_0}{2}(2\mu T_2 - T_1^2 - \mu^2)\right] - A_0\Bigg\}$$

Therefore, T^* is the best cycle length.

Theorem: It is obvious that $E < 0$

Proof

$d_1 < D_0, \ \ \alpha_1 < \alpha_2, \ r_1 < r_2, \ \ T_1 < T_2, \ \ T_1 \le \mu \le T_2, \ \ I_d > d_1,$

$E < S\left[d_1\alpha_1(1-r_1)T_1 - \frac{d_1\alpha_1(1-r_1)}{2}[3T_1^2 + T_1^2]\right] - (I_d + d_1\alpha_1 T_1)C - h\left[\frac{d_1\alpha_1 {T_1}^2}{2} + T_1 I_d + d_1\alpha_1\left[-\frac{{T_1}^2}{\theta} - \frac{T_1^2}{2\theta} + \frac{T_1^2}{\theta} + \frac{T_1}{\theta} - \frac{{T_1}^2}{2\theta}\right] + I_d(T_1 - T_1)\right] - C_0\left[I_d + \frac{d_1}{2}\left(T_1^2 - 2{T_1}^2 + {T_1}^2\right)\right] - A_0$

$$= S\left[d_1\alpha_1(1-r_1)[T_1 - 2T_1^2]\right] - (I_d + d_1\alpha_1 T_1)C - h\left[\frac{d_1\alpha_1 {T_1}^2}{2} + T_1 I_d + d_1\alpha_1\left[\frac{{T_1}^2}{2} - \frac{T_1^2}{\theta} + \frac{T_1}{\theta}\right]\right] - C_0 I_d - A_0$$

$$= -ST_1 d_1\alpha_1(1-r_1)[2T_1 - 1] - (I_d + d_1\alpha_1 T_1)C - h\left[d_1\alpha_1\left[\frac{{T_1}^2\theta - T_1^2 + 2T_1}{2\theta}\right] + T_1 I_d\right] - C_0 I_d - A_0$$

$$= -ST_1 d_1\alpha_1(1-r_1)[2T_1 - 1] - (I_d + d_1\alpha_1 T_1)C - h\left[d_1\alpha_1\left[\frac{T_1(T_1\theta - 2T_1 + 2)}{2\theta}\right] + T_1 I_d\right] - C_0 I_d - A_0$$

$$= -ST_1 d_1\alpha_1(1-r_1)[2T_1 - 1] - (I_d + d_1\alpha_1 T_1)C - h\left[d_1\alpha_1\left[\frac{T_1\theta}{2\theta}\right] + T_1 I_d\right] - C_0 I_d - A_0$$

$E = -ST_1 d_1\alpha_1(1-r_1)[2T_1 - 1] - (I_d + d_1\alpha_1 T_1)C - h\left[d_1\alpha_1\left[\frac{T_1\theta}{2\theta}\right] + T_1 I_d\right] - C_0 I_d - A_0 < 0$

Where $r_1 <<< 0$

Therefore

$$E = -\left(ST_1 d_1\alpha_1(1-r_1)[2T_1] + (I_d + d_1\alpha_1 T_1)C + h\left[d_1\alpha_1\left[\frac{T_1\theta}{2\theta}\right] + T_1 I_d\right] + C_0 I_d + A_0\right) < 0$$

SUFFICIENCY CONDITION

To find the sufficiency condition, we take the second derivative of equation (9) with respect to T and simplify to have:

$$\frac{d^2PF}{dT^2} = -\frac{2}{T}\left\{\frac{dPF}{dT}\right\} - \frac{1}{T^2}D_0\mu[2T](S_1 - h)$$

$\frac{d^2PF}{dT^2} = -\frac{2}{T}\frac{dPF}{dT}\Big|_{T^*} - \frac{1}{T^2}D_0\mu[2T] \qquad (S_1 - h)$

Since $T^*$is the optimal then, $\frac{dPF}{dT}\Big|_{T^*} = 0$

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$$\frac{d^2PF}{dT^2} = -\frac{2}{T}D_0\mu(S_1 - h) < 0 \qquad [S_1 > h] \qquad (11)$$

Solution Algorithm

In order to obtain the optimal solution of the inventory system, we propose the following algorithm:

Step I: Start

Step II: Input the appropriate values of the system parameters.

Step III: Using Mats lab Mathematical software solve equation (10), $\frac{dPF}{dT} = 0$ and find T.

Step IV: Compare the obtained value of T from step III with μ.

(i) If $T < \mu$ then T is feasible solution. Evaluate $\frac{d^2PF}{dT^2} < 0$, the value of $T = T^*$ is the optimal value. Then go to step V.

(ii) If $T > \mu$ then T is infeasible. Choose another set of parameters and repeat step III to IV.

Step V: Substitute $T = T^*$in equations (12) and (13) to obtain the corresponding optimal total profit $(P)$and optimal order quantity, Q respectively.

Step VI: Consider $I_b(t)$ as complete backlogging

Step VII: End.

4. Numerical Example

Consider the inventory system with the following input parameters:

Let Hc= 30, D_0 = 20000, μ=0.25, $S_1 = 40$, S=50, $d_1 = 10000$, C=40, $I_d = 10000$, $C_0 = 30$, $A_0 = 500$, $\theta = 0.005$, r_1=0.01, r_2 =0.02, T_1=2, T_2=3, $\alpha_1 = 0.01, \alpha_2 = 0.02$ and substitute the values in equations, we found the values of T*=0.63 (year) ≈ 230 days, $I_0 = 30000$, (Units), PF=282109.5 (N).

Sensitivity Analysis

Parameters	% change in parameter	Change in I_0	Change in T^*	change in PF
	20	15.5907	0.7143	230292.17
	5	15.5907	0.7870	243822.94
H	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8424	252859.92
	-20	15.5907	0.9408	266758.67
	20	15.5907	0.8129	298188.20
	5	15.5907	0.8136	260786.68
D_0	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8142	235872.33
	-20	15.5907	0.8154	198500.79

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	20	15.5907	0.9369	317467
	5	15.5907	0.9612	275089.74
M	0	15.5907	0.8139	248328.46
	-5	15.5907	0.4512	1448328.46
	-20	15.5907	0.4627	145271.31
	20	15.5907	0.8700	284723.36
	5	15.5907	0.8283	257414.49
S	0	15.5907	0.88138	248329.51
	-5	15.5907	0.7991	239225.10
	-20	15.5907	0.7527	212032.86
	20	16.7088	0.8139	248339.54
	5	15.5907	0.8139	248332.01
d_1	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8140	248327
	-20	15.5907	0.8139	248319.48
	20	15.5907	0.8139	248329.35
	5	15.5907	0.8139	248329.47
r_1	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8139	248329.55
	-20	15.5907	0.8139	248329.67
	20	15.5907	0.8106	248331.09
	5	15.5907	0.8131	248330.15
r_2	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8147	248328.69
	-20	15.5907	0.8172	248325.28
	20	15.5907	0.4532	248131.31
	5	15.5907	0.4953	162667.73
T_1	0	15.5907	0.8135	248272.55
	-5	15.5907	0.9352	265389.53
	-20	15.5907	0.6949	265098.22
	20	15.5907	0.8147	248228.80
	5	15.5907	0.8141	248304.33
I_d	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8137	248354.68
	-20	15.5907	0.8131	248430.20
	20	15.5907	0.8144	248250.79
	5	15.5907	0.8140	248309.83
C	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8138	248349.19
	-20	15.5907	0.8133	248408.23
	20	15.5907	0.8497	251908.54

ISSN 2734-3839 (Print), ISSN 2735-962X (E-Copy)
http://www.journal.kasu.edu.ng/index.php/kjms DOI/10.5281/zenodo.12627471.svg

	5	15.5907	0.8243	249428.08
θ	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8022	247036.40
	-20	15.5907	0.7564	241318.55
	20	15.5907	0.8636	279935.95
	5	15.5907	0.8266	256220.92
C_o	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8009	240443.20
	-20	15.5907	0.7604	216800.04
	20	15.5907	0.8144	248257.38
	5	15.5907	0.8140	248311.48
A_o	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8138	248349.54
	-20	15.5907	0.8134	248401.64

From the table above we see that;

The % values of h, D_o, μ, S, T_1, C, C_o, and A_o of I_o values remain on change.

The higher % values of d_1, r_1, r_2, I_d, of I_o values increase.

The lower % values of d_1, r_1, r_2, I_d, of I_o values decrease.

The higher % values of h, D_O, μ, d, r_1, of T^* values decrease.

The higher % values of S,I_d,r_2,C_O,,C, A_O,θ of T^* values increase.

The lower % values of h, D_O, μ, d_1, r_1, T_1 of T^* values increase.

The lower % values of S,I_d,r_2,C_O,,C, A_O,θ of T^* values decrease.

The higher % values of h,r_2,I_d,C, θ,A_O of PF values decrease.

The higher % values of D_O,μ, S,r_1,d_1,C_0 ,T_1 of PF values increase

The lower % values of h,r_2,I_d,C, θ,A_O of PF values increase.

The lower % values of D_O,μ, S,r_1,d_1,C_0 ,T_1 of PF values decrease

5. Conclusion

In this work, we introduce a mathematical model for the inventory of non-instantaneous, delayed deteriorating items with ramp-type demand. When a consumer goods brand is introduced to the market, the demand for its first rises linearly until the market stabilizes and the demand remains constant until the conclusion of the inventory cycle. This is known as the ramp type demand rate. A unit selling price discount is offered for products that begin to deteriorate after a specific amount of time, with components stock dependent on demand both before and after degradation. Demand is partially stock dependent and partially stock and selling price dependent when a discount is offered.

Complete backlog and pre and post degradation discounts on the unit selling price are allowed by the mathematical model that has been devised. Then, the permitted amount of both types of discounts greatly increases demand while maximizing profit. Additionally, the manager can use the suggested model to help precisely calculate the ideal order amount and average cost per unit. Additionally, the suggested approach can be used to inventory control of some decaying products, like food. Realistic hypotheses like probabilistic demand as a finite rate of replenishment could be included in subsequent research.

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