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Optical Soliton Solutions of Burgers-Fisher and Burgers-Huxley Equations

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Abstract
In this study, we investigate the soliton solutions of two prominent nonlinear partial differential equations: the Burgers-Fisher equation and the Burgers-Huxley equation. These models are instrumental in describing a variety of physical, biological, and ecological phenomena where both nonlinear advection and reaction-diffusion mechanisms play significant roles. To derive the soliton solutions of these equations, we employ the Bernoulli sub-ODE method, a powerful analytical technique for solving nonlinear ODEs. This method transforms the original partial differential equations into more tractable forms, facilitating the discovery of soliton solutions. Our findings demonstrate the existence of stable, localized wave solutions for both equations, providing deeper insights into the wave dynamics governed by these models.

Keywords: Soliton solution, Bernoulli Sub–ODE Method, Burger–Fisher equation and Burger–Huxley Equation.

1. Introduction

In the study of nonlinear partial differential equations, soliton solutions hold a significant place due to their remarkable properties and wide-ranging applications in various fields of science and engineering (Wadati, 2001). Solitons are self-reinforcing solitary waves that maintain their shape while traveling at constant speeds, resulting from a delicate balance between nonlinear and dispersive effects in the medium (Malfliet, (1992); A.G Johnpillai, 2013). We are going to consider two well-known models that exhibit soliton solutions are the Burgers-Fisher and Burgers-Huxley equations.

Burgers-Fisher Equation: The Burgers-Fisher equation combines features of both the Burgers equation and the Fisher-KPP (Kolmogorov-Petrovsky-Piskunov) equation. The Burgers equation is a fundamental partial differential equation in fluid mechanics and traffic flow, known for describing viscous fluid flow and shock waves (A. Veksler, 2005). The Fisher-KPP equation, on the other hand, is a reaction-diffusion equation used to model the spread of advantageous genes in a population. The equation that shows a prototypical model for describing the interaction between

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the reaction mechanism, convection effect, and diffusion transport. It arises in the field of gas dynamic, traffic flow and physics application. The equation is given in the form (Wazwaz A. , 2008).

$$u_t = u_{xx} + uu_x + u(1-u) \quad (1)$$

Where $u(x,t)$ represents the space variable. This equation models phenomena where both nonlinear advection and reaction-diffusion mechanisms are present, such as in population dynamics, chemical reactions, and ecological systems. The interplay between the nonlinear advection term uu_x and the diffusive term u_{xx} leads to rich and complex behaviors, including the formation of solitons (X.F Yang, 2015; W Zhao M Munir, 2015).

Burgers-Huxley Equation: The Burgers-Huxley equation similarly integrates the dynamics of the Burgers equation with those of the Huxley equation, which is another reaction-diffusion model (Wazwaz A. , (2005)). The Huxley equation is utilized to describe nerve pulse propagation and various excitable systems. The equation describe a wide class of physical nonlinear phenomena such as interaction between reaction mechanism convection effect and diffusion transport. The equation is given in the form (Wazwaz A. , 2008)

$$u_t = u_{xx} + uu_x + u(k-u)(u-1) \quad (2)$$

Where k is a parameter that controls the bistable nature of the reaction term. This equation is significant in the study of nerve conduction, chemical kinetics, and pattern formation (A.G Johnpillai, 2013). Numerous Investigation have been made to examine the solution of the equations using different method such as: Wadati trace Method (Wadati, 2001), Inverse scattering method (Wazwaz A. , (2005)), Backlund transformation method, the tanh Method (W Zhao M Munir, 2015), the tanh–Coth method (Wazwaz A. , (2005))and lots more methods. This research aims to explore the soliton solutions of the Burgers-Fisher and Burgers-Huxley equations, analyze their properties, and investigate the conditions under which these solutions arise. By doing so, we can gain a better understanding of the complex dynamics modeled by these equations and their applications in real-world scenarios.

2. Description of Method

Consider the PDE given by (A. R. Alharbi, (2020); X.F Yang, 2015)

$$P(u, u_t, u_x, u_{xt}, u_{xx\cdots}) = 0 \quad (3)$$

Where $u(x,t) = u(\xi)$

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Step 1:

Using the travelling transformation $u(x,t) = u(\xi)$ where $\xi = x \pm vt$ Eq. (3) can be transformed into the following ODE

$$P(u', u'', u''', \ldots\ldots) = 0 \tag{4}$$

With $u(\xi) = \frac{\partial u}{\partial \xi}$

Step 2:

Suppose that Equation (4) has a solution of the form

$$\mathrm{u}(\xi) = \sum_{i=0}^{n} a_i G^i \tag{5}$$

Where $\mathrm{G} = \mathrm{G}(\xi)$ satisfied the equation

$$G^i + \lambda \mathrm{G} = \mu G^2 \tag{6}$$

$a_i(-\mathrm{n} \leq \mathrm{i} \leq \mathrm{n})$ are constant and $\mu \neq 0, \lambda \neq 0$.

Equation (6) is a type of Bernoulli equation having the following solution

$$\mathrm{G} = \frac{-\lambda}{2\mu}(\mathrm{Tanh}\,[\frac{\lambda\xi}{2}] - 1) \tag{7}$$

$$\mathrm{G} = \frac{-\lambda}{2\mu}(\mathrm{Cosh}\,[\frac{\lambda\xi}{2}] - 1) \tag{8}$$

Step 3:

The positive integer n can be obtained by balancing the highest-order derivatives and the highest order nonlinear term that appears in the equation (4).
The balancing formula is given as

$$\mathrm{D}\frac{d^a u}{d\xi^a} = n + a,\ \mathrm{D}(u^b(\frac{d^a u}{d\xi^a})^c = bn + c(n+a) \tag{9}$$

Thus we can obtained the value of n in equation (4) by using equation (9).

Step 4:

Substitute equation (5) into (4), using (6) and collect all terms of the same power of$\mathrm{G}(\xi)$. Setting each coefficient of G^i zero we obtain a system of algebraic equation, solving the equation we obtain the value of a_i and other parameters.
Lastly substituting the values of a_i and the other parameters in equation (5),
We get the solution of equation (3).

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3. Application of Method.

3.1 Burger–Fisher Equation.

Using the transformation $u(x,t) = u(\xi)$ where $\xi = x - ct$ (S. Z Hassan, 2018) Eq. (1) can be transformed into the following ODE:

$$cu' + uu' + u'' + u - u^2 = 0 \quad (10)$$

Balancing u'' and u^2 in equation (10), we get $n = 2$. Therefore equation (5) can be written as

$$u(\xi) = a_0 + a_1 G^1 + a_2 G^2 \quad (11)$$

Where a_0 , a_1 , a_2 are non-zero constants to be determine later. Substituting equation (11) and its derivatives into (10) and collecting all the terms with the same power of G together, and equating to zero, we obtain the following system of algebraic equation:

G^0 :

$$-(a_0 - 1)a_0 = 0, \quad (12)$$

G^1 :

$$a_1(-a_0(\lambda + 2) - c\lambda + \lambda^2 + 1) = 0, \quad (13)$$

G^2 :

$$a_2(-2a_0(\lambda + 1) - 2c\lambda + 4\lambda^2 + 1) + a_1\mu(a_0 + c - 3\lambda) + a_1^2\big(-(\lambda + 1)\big) = 0, \quad (14)$$

G^3 :

$$2a_2\mu(a_0 + c - 5\lambda) + a_1\big(2\mu^2 - a_2(3\lambda + 2)\big) + a_1^2\mu = 0, \quad (15)$$

G^4 :

$$-a_2(a_2(2\lambda + 1) - 3a_1\mu - 6\mu^2) = 0, \quad (16)$$

G^5 :

$$a_2^2\mu = 0, \quad (17)$$

Solving the above system of equations from (12)-(17), we obtain the following set of Values:

$$a_0 = 1; a_1 = 2\mu; a_2 = 0 ; c = \frac{-5}{2}; \ \lambda = \frac{-1}{2}$$

Substituting Set of values into equation (11), we get the following Solutions.

$$u(\xi) = \frac{1}{2}(\mathrm{Coth}\left(\frac{1}{8}(5t + 2x)\right) + 1) \quad (18)$$

And

$$u(\xi) = \frac{1}{2}(\mathrm{Tanh}\left(\frac{1}{8}(5t + 2x)\right) + 1) \quad (19)$$

Where $\xi = x \pm ct$ implies that $\xi = x + \frac{5}{2}t$.

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3.2 Burger-Huxley Equation.

Using the transformation $u(x,t) = u(\xi)$ where $\xi = x - ct$ (S. Z Hassan, 2018) Eq. (1) can be transformed into the following ODE:

$$c\,u' + u\,u' + u'' + u(k - u)(u - 1) = 0 \qquad (20)$$

Balancing u'' and u^3 in equation (20), we get $n = 1$. Therefore equation (5) can be written as

$$u(\xi) = a_0 + a_1 G^1 \qquad (21)$$

Where a_0 , a_1 are non-zero constants to be determine later. Substituting equation (21) and its derivatives into (20) and collecting all the terms with the same power of G together, and equating to zero, we obtain the following system of algebraic equation:

G^0 :

$$(a_0 - 1)a_0\,(k - a_0) = 0, \qquad (22)$$

G^1 :

$$a_1(a_0(2k - \lambda + 2) - 3a_0^2 + \lambda(\lambda - c) - k) = 0, \qquad (23)$$

G^2 :

$$a_1\,(a_1(k - \lambda + 1) + a_0(\mu - 3a_1) + \mu(c - 3\lambda)) = 0, \qquad (24)$$

G^3 :

$$a_1(a_1\,\mu - a_1^2 + 2\mu^2) = 0, \qquad (25)$$

Solving the above system of equations from (22)-(25), we obtain the following set of Values:

Set 1:

$$a_0 = 1;\ \lambda = \frac{-1}{2};\ a_1 = 2\mu\,;\ c = -2k - \frac{-1}{4}$$

Set 2:

$$a_0 = k;\ \lambda = -2\lambda;\ a_1 = 2\mu\,;\ c = \frac{-k^2 + \lambda^2 - \lambda k + k}{\lambda}$$

Set 3:

$$a_0 = k;\ k = \lambda + 1;\ a_1 = -\mu\,;\ c = \frac{-k^2 + \lambda^2 - \lambda k + k}{\lambda}$$

Set 4:

$$a_0 = 1;\ k = 1 - \lambda;\ a_1 = -\mu\,;\ c = \frac{\lambda^2 - \lambda + k - 1}{\lambda}$$

Set 5:

$$a_0 = 0;\ K = 2\lambda;\ a_1 = 2\mu\,;\ c = \frac{\lambda^2 - k}{\lambda}$$

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Set 6:

$$a_0 = 1;\ \mathrm{k} = 2\lambda + 1; a_1 = 2\mu\ ; \mathrm{c} = \frac{\lambda^2 - \lambda + \mathrm{k} - 1}{\lambda}$$

Substituting Set 1 of values into equation (21), we get the following Solutions.

$$u_{1,1} = \frac{1}{2}(1 - \mathrm{Coth}\ \left(\frac{1}{4}(2\left(k - \frac{1}{4}\right)t + x\right))) \tag{26}$$

And

$$u_{1,2} = \frac{1}{2}(1 - \mathrm{Tanh}\ \left(\frac{1}{4}(2\left(k - \frac{1}{4}\right)t + x\right))) \tag{27}$$

Where $\xi = x - ct$ implies that $\xi = x + (2\mathrm{k} - \frac{1}{4})t$.

Substituting Set 2 of values into equation (21), we get the following Solutions.

$$u_{2,1} = \frac{1}{2}(\mathrm{Coth}\ \left(\frac{1}{8}(-3t - 2\mathrm{x})\right) + 1) \tag{28}$$

And

$$u_{2,2} = \frac{1}{2}\left(\mathrm{Tanh}\ \left(\frac{1}{8}(-3t - 2\mathrm{x})\right) + 1\right) \tag{29}$$

Where $\xi = x - ct$ implies that $\xi = x - (\frac{-\mathrm{k}^2 + \lambda^2 - \lambda\mathrm{k} + \mathrm{k}}{\lambda})t$.

Substituting Set 3 of values into equation (21), we get the following Solutions.

$$u_{3,1} = \frac{1}{4}(\mathrm{Coth}\ \left(\frac{1}{8}(3t + 2\mathrm{x})\right) + 3) \tag{30}$$

And

$$u_{3,2} = \frac{1}{4}\left(\mathrm{Tanh}\ \left(\frac{1}{8}(3t + 2\mathrm{x})\right) + 3\right) \tag{31}$$

Where $\xi = x - ct$ implies that $\xi = x - (\frac{-\mathrm{k}^2 + \lambda^2 - \lambda\mathrm{k} + \mathrm{k}}{\lambda})t$.

Substituting Set 4 of values into equation (21), we get the following Solutions.

$$u_{4,1} = \frac{1}{4}(\mathrm{Coth}\ \left(\frac{1}{8}(5t + 2\mathrm{x})\right) + 5) \tag{32}$$

And

$$u_{4,2} = \frac{1}{4}\left(\mathrm{Tanh}\ \left(\frac{1}{8}(5t + 2\mathrm{x})\right) + 5\right) \tag{33}$$

Where $\xi = x - ct$ implies that $\xi = x - (\frac{\lambda^2 - \lambda + \mathrm{k} - 1}{\lambda})t$.

Substituting Set 5 of values into equation (21), we get the following Solutions.

$$u_{5,1} = \frac{1}{2}(\mathrm{Coth}\ \left(\frac{1}{8}(-5t - 2\mathrm{x})\right) - 1) \tag{34}$$

And

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$$u_{5,2} = \frac{1}{2}\left(\mathrm{Tanh}\ \left(\frac{1}{8}(-5t - 2\mathrm{x})\right) - 1\right) \quad (35)$$

Where $\xi = x - ct$ implies that $\xi = x - (\frac{\lambda^2 - \mathrm{k}}{\lambda})t$.

Substituting Set 6 of values into equation (21), we get the following Solutions.

$$u_{6,1} = \frac{1}{2}(\mathrm{Coth}\ \left(\frac{1}{8}(t - 2\mathrm{x})\right) + 1) \quad (36)$$

And

$$u_{6,2} = \frac{1}{2}\left(\mathrm{Tanh}\ \left(\frac{1}{8}(t - 2\mathrm{x})\right) + 1\right) \quad (37)$$

Where $\xi = x - ct$ implies that $\xi = x - (\frac{\lambda^2 - \lambda + \mathrm{k} - 1}{\lambda})t$.

4. Discussion of Obtained Results

In this study, we have successfully derived soliton solutions for both the Burgers-Fisher and Burgers-Huxley equations using the Bernoulli sub-ODE method. The solutions are expressed in terms of hyperbolic functions: coth (hyperbolic cotangent) and tanh (hyperbolic tangent). These specific forms of solutions provide deep insights into the nature and behavior of the solitons in these nonlinear systems.

The soliton solutions expressed in terms of coth and tanh functions are significant because they provide explicit, analytical forms that describe the behavior of nonlinear waves in the Burgers-Fisher and Burgers-Huxley models. These solutions offer clear insights into how system parameters influence wave profiles, including their amplitude, width, and stability.

Coth Solutions: These solutions feature sharp transitions between two states, resembling shock waves. The hyperbolic cotangent function diverges as its argument approaches zero, indicating a steep, almost discontinuous change in the space variable **u**. This behavior is typical of systems with strong nonlinear advection, where wave fronts are abrupt and propagate rapidly. Such solutions are pertinent to scenarios involving strong nonlinear interactions and bistable reactions, where the system quickly switches between states. The hyperbolic cotangent (coth) solutions are characterized by steep gradients and a highly localized nature, making them particularly useful for modeling sharp interfaces or fronts in reaction-diffusion systems.

Tanh Solutions: The hyperbolic tangent solutions describe smoother, sigmoidal transitions between two states. Unlike coth, the tanh function approaches finite values asymptotically, representing stable, localized waves that maintain their shape and speed over time. These solutions are relevant for describing phenomena where the reaction-diffusion term plays a significant role, leading to gradual changes. The hyperbolic tangent solutions for the Burgers-Huxley equation represent smooth, localized waves, applicable to excitable systems such as nerve pulse propagation, where the field variable changes gradually and predictably over space and time. In

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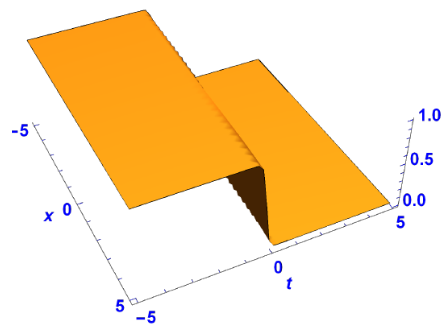
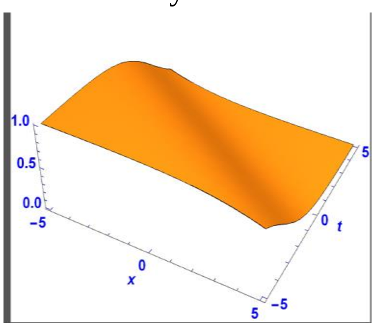
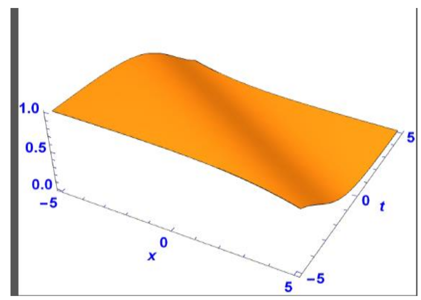


contrast, hyperbolic tangent (tanh) solutions are smoother and describe more gradual transitions, suitable for systems where changes are continuous and less abrupt.





4.1 Physical Interpretation of Solution.

This section, gives the dynamics behaviors of the obtained solution of Burger–Fisher equation and Burger– Huxley Equation considering appropriate values of parameters.

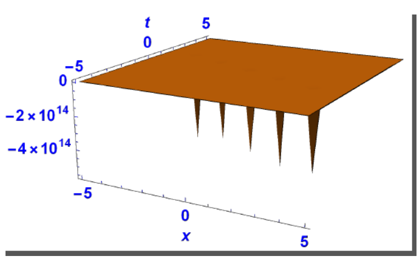


Fig. 1a

Fig. 1a

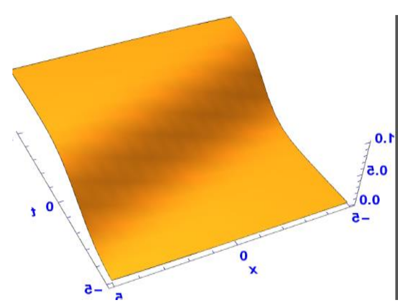


Fig. 1b

Figure 1: Figure 1(a) and 1(b) gives dynamics behaviors of Solutions obtained in Eq. (18) and Eq. (19) by considering the range values of -5 ≤ x ≤ 5, -5 ≤t ≤5.

Fig. 2a

Fig. 2b

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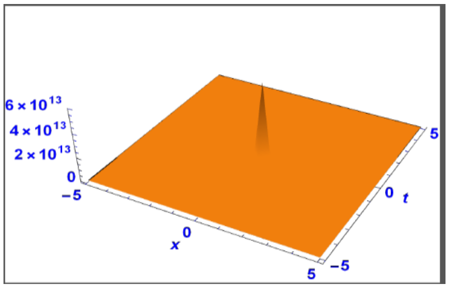
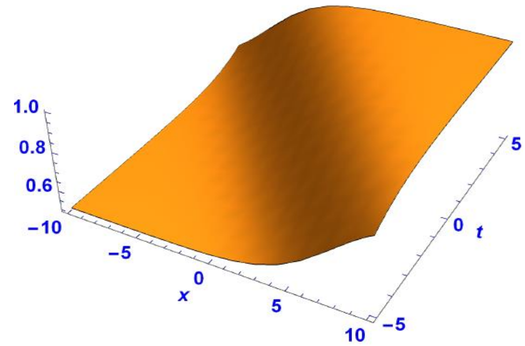
Figure 2: Figure 2(a) and (2b) gives dynamics behaviors of Solutions obtained in Eq. (26) and Eq.





(27) by considering the values $k = 15$, -5 ≤ x ≤ 5, -5 ≤t ≤5.

Fig. 3a

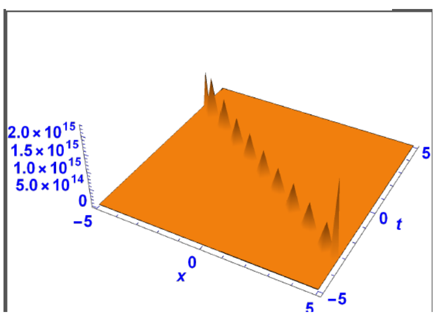
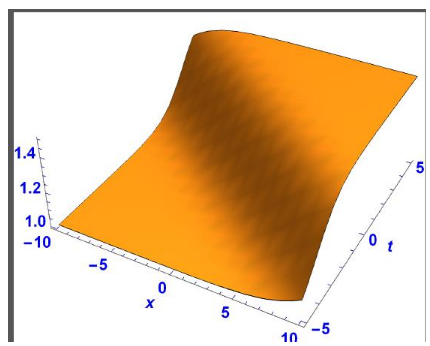
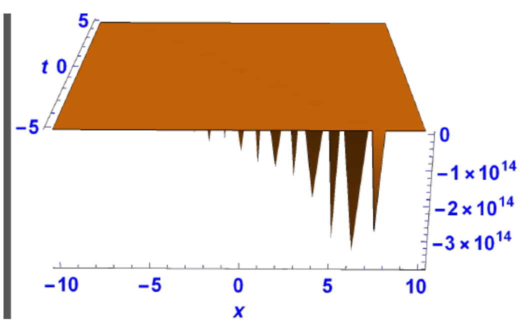


Fig. 3b





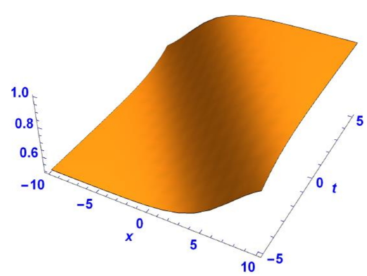


Figure 3: Figure 3(a) and 3(b) gives dynamics behaviors of Solutions obtained in Eq. (28) and Eq. (29) by considering the range values of -5 ≤ x ≤ 5, -5 ≤t ≤5.

Fig. 4a

Fig. 4b

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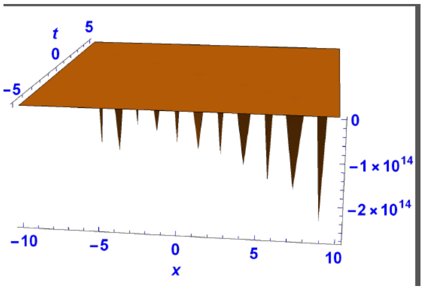
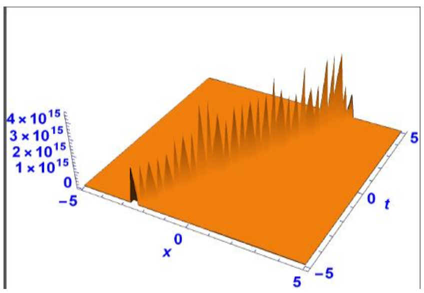


Figure 4: Figure 4(a) and 4(b) gives dynamics behaviors of Solutions obtained in Eq. (30) and Eq. (31) by considering the range values of $-5 \leq x \leq 5$, $-5 \leq t \leq 5$.



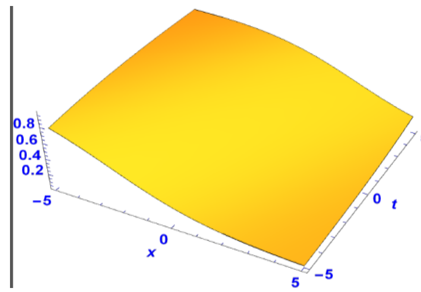


Fig. 6a **Fig. 6b**

Figure 6: Figure 6(a) and 6(b) gives dynamics behaviors of Solutions obtained in Eq. (36) and Eq. (37) by considering the range values of $-5 \leq x \leq 5$, $-5 \leq t \leq 5$.

5. Conclusion

The application of the Bernoulli sub-ordinary differential equation (ODE) method to the Burgers-Fisher and Burgers-Huxley equations has yielded significant results, providing explicit soliton solutions that enhance our understanding of these complex nonlinear systems. The derived solitons exhibit stability and localization, maintaining their shape and speed over time, which is characteristic of soliton behavior. For the Burgers-Fisher equation, the soliton solutions illustrate the balance between nonlinear advection and reaction-diffusion mechanisms, offering insights into phenomena such as population dynamics and chemical reactions. Similarly, the soliton solutions for the Burgers-Huxley equation highlight the dynamics of excitable systems and nerve pulse propagation. The obtained soliton solutions have broad implications across various scientific and engineering disciplines. In fluid dynamics, they model shock waves and viscous flows, while in population genetics, they describe the spread of advantageous genes. In neuroscience, the Burgers-Huxley solitons model nerve pulse propagation, contributing to the development of more accurate models of neural activity

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