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Using Matlab PDE Tool Box on Solutions of High Pressure Transient Gas flow in Pipeline

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Abstract
In finding solutions to transient flow the accurate prediction is highly needed. Compress gas are on high pressure. Using PDEtoolbox the boundary conditions are treated numerically. From Graphic User Interface (GUI) MATLAB there is always window for default boundary values g and b which are exported into M-file for computation. This method solution enable us to bring the domain of boundary condition and governing equation on nonlinear systems of partial differential equations for simpler and effective computation. From the result obtained will represent real life situation of the system and improved the accuracy and computational time when compared with previous published results.

1. Introduction

Natural gas has been fossil fuels used by big engines in large industries and many researches are conducted on possibility of achieving this globally impact of mixture with hydrogen gas. The mixture of hydrogen with natural gas is starting point toward full fledge hydrogen usage as energy source (Haeseldonckx and D'haeseleer, 2011). Hydrogen Natural Gas Mixture (HNGM) can be used in engine and turbines since hydrogen is also a flammable gas (Cheng R et al., 2009, Tabkhi F et al., 2008). Energy from natural gas source is one of with less emission, but has high carbon emission when leakage occurs. It also has low burning capacity. For that the overcoming of problems associated to natural gas leakage need to be considered for effective performance and reduction in carbon emission. The inclusion of hydrogen to natural gas has numerous, benefit on the reduction of greenhouse gas level (Yang and Ogden, 2007,B. G, Agaie,2014). With hydrogen having low burning capacity than natural gas, the mixture will create storage and transportation problem when the two gasee are mixed. But the problem of high emission when leakage occur of natural gas will be taken care off. Hydrogen been the lightest and most abundant flammable gas in the universe is receiving global attention as an alternative energy source. It has high burning capacity but poor storage.(Elaoud and Hadj-Taı¨eb, 2008). In an attempt towards realizing hydrogen economy, natural gas are enriched with hydrogen as a starting point (Baufume et al., 2011, Mohammadi et al., 2012). The global warming increasing rate of due to carbon emission from energy source is increasing as a result of energy needs increase therefore, alternative source is required. World energy continuously increases which is expected to rich 60% of 2005 by 2030. The present energy source is mostly from fossil fuel therefore there is need of less or non-carbon emission source. In 19th century coal dominate energy source which was followed by petroleum oil, natural gas and nuclear energy takes over in 20th and 21st centuries. There is clear indication that hydrogen will possible taking over before the end of 21st century due to recent development (Winter, 2009). Due to research on possible replacement of fossil fuel sources of energy toward saving the greenhouse gas. To address problem of low burning capacity associated with natural gas and poor storage capacity of hydrogen, researches are conducted for possibility of mixing the two gases. Hence natural gas is enriched by adding some percentage of hydrogen, to obtain hydrogen compressed natural gas mixture (HCNG). HCNG needs to be transported to consumption point and the easiest way is through pipeline. The transportation of gas is normally at unsteady state due to rapid closure of valves in meeting customer demand. To reduce production cost the existing natural gas pipeline network is normally used consequently there is need for proper analysis of HCNG transportation. HCNG will also serve as a testing ground toward free emission energy source by 2030. Different types of models and numerical method are previously presented amid at improving accuracy and reduce computational cost; some basic equations are sometimes neglected based on research area of interest. Transient flow models of some selected problem was investigated (Kessal, 2000). The simulation natural gas flow in pipeline was investigated by many researchers where temperature effect is neglected with governed by conservation forms of mass and momentum. A new method was presented by (Zhou and Adewumi, 1996) on horizontal natural gas flow pipe, in which the lost accuracy due to neglected term are recovered as shown in presented results. Non isothermal condition on slow and fast transient flow of high pressure of gases at overestimation was observed for a steady state condition assumption on heat transfer by (Chaczykowski, 2010).

Various methods are previously present all to improve on accurate prediction of flow parameters. Fully implicit finite difference method base on Newton-Raphson method in calculating transient flow behavior of natural gas was investigated where greater reduction on computation time is achieved (Kiuchi, 1994). Reduce order Model was also introduced on pipeline problem which show increase on accurate flow parameter prediction and reduce computational time (B.G.Agaie, 2014). The aging of pipes sometime leads to leakage or failure due to corrosion or rupture. A sensor Pipe Net system was developed to detect leakage and failure so as to reduce the millions of dollar spends on monitoring water pipeline (Stoianov et al., 2007). Similar sensor can also be used in gas pipeline. Two monitoring methods are developed for location of leak pipe distance from transient flow simulation which demonstrate significant accuracy in detecting and locating of pipe leakage (Jang et al., 2010). The two methods are cross correlation monitoring and pressure differential monitoring methods. Robustness and mechanical resistance research on natural gas can be extended to HNGM. The thermodynamics properties of the mixture will be different there is the need for proper investigation in the transportation of HNGM during transient regime of mixture (Elaoud and Hadj-Taı¨eb, 2008, B.G. Agaie, 2014). The numerical investigation of high

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pressure hydrogen natural gas mixture was presents by(Elaoud and Hadj-Taı¨eb, 2008) where pressure and velocity dependent variable are analysis. The thermal condition of hydrogen natural gas mixture was presented where hydrogen is injected into transient natural gas transient. By using the predictive Soave-Redlich-Kwong method(Uilhoorn, 2009). More research on transportation, production and storage capacity of HNGM is required to meet the increasing rate of its usage for increasing world energy source economy. This can be achieved mostly for improve on transportation system of HNGM. Although it will take decades before significant impact is made on the world economic (Wang, 2011, Agaie and Garba, 2020). Hydrogen and hydrogen natural gas mixture (HCNG) needed high pressure for effective transportation and therefore requires accurate prediction of expected pressure and velocity. The transient occur during flow due to rapid closure and opening of valves will affect consumer demands. With the present of high capacity computer and related computational programming languages such as MATLAB, which can be used in analyzing transient behavior of gas flow. The PDEtoolbox in MATLAB can be used on prediction flow parameters of HCNG transient flow parameter. The MATLAB tool is expected to reduce the computational time cost as well as improve computational accuracy.

2. Numerical Technique

The numerical technique contributes significantly impact on finding solutions to various problems. For optimal operation of gas flow can be accurately represented and that will increase the accuracy on pressure drop prediction. The efficiency and accuracy of natural transient flow using transient was investigated.(Behbahani-Nejad and Bagheri, 2010, Behbahani-Nejad and Shekari, 2010). Many mathematical models are continuously developed by researchers where different methods are used aimed at improvement of accuracy in flow parameters. It has be proven that one dimensional flow model is sufficient in the analysis of transient flow of natural gas flow. In the research conducted by(Thorley and Tiley, 1987)a literature review was excellently carried out on transient flow problems solution methods. Where characteristic method has shown an advantage on discontinuous flow cases, such as leakage or sudden cut, but relatively slow. The method with less accuracy is the explicit finite difference methods on transient flow problems, while Galerk infinite element is not commonly use in gas flow problems due to it computational storage demand .(Abbaspour et al., 2010). The Reduce Order Modelling (ROM) technique has been demonstrating high degree of accuracy in computation fluid problems and also have less computational time when compare to other numerical methods. Usually achieved by transforming a large number of primitive variables into smaller amount at decoupled equations (Romanowski and Dowell, 1997). The ROM is the constructed used Eigen analytical therefore few nodes are retain from the original nodes. The ROM is often used in aerodynamic problem but was introduced to gas flow problem and significant accuracy was presented. The analysis of aerodynamic problem, design of the wing fuselage system and exhibited a low cost in computation of ROM(Jun et al., 2010). It also can be applied in analysis of subsonic flow. From the concept of computation fluid dynamics equations that are mostly involve in fluids analysis are the Euler and the navies stokes equations. The conservation forms of these equations are usually nonlinear hyperbolic partial

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differential equation which cannot be solve using the MATLAB command code PDENONLIN.(Kaist, 2012).MATLAB Simulink demonstrated efficiency on natural gas transient flow on pipeline network.

3. Problem Formulation

The transportation of gas is governed by principle of conservation laws, however mass conservation equation and momentum will be involved while energy lost is ignore making our problem then on-isothermal. Therefore the problem shall be governed by two equations. Since the isothermal condition is assumed the polytrophic process with be assumed to be at homogenous mixed. For that inviscid fluid condition is sufficient to define our flow problem and hence one dimensional Euler equation will be sufficient in the analysis (Zhou and Adewumi, 1996, Osiadacz and Chaczykowski, 1998).

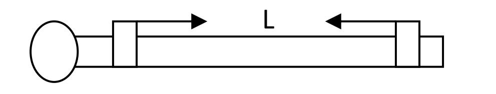

Figure1: Pipe geometry

Assuming an inviscid fluid of gas mixture, then the transient flow behavior will be one dimensional conservation forms of Euler`s continuity and momentum equations neglecting the body force.

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \tag{1}$$

For $p=\rho c^2$, where c is the fluid celerity wave the require momentum equation shall be given as (Behbahani-Nejad & Shekari, 2010)

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2+c^2\rho)}{\partial x}+\frac{f\rho u|u|}{2d}=0 \tag{2}$$

where ρ and u are fluid mixture density, velocity and f pipe fraction factor respectively,

Initial conditions:

$$\frac{\partial \rho(x,0)}{\partial x}=0 \tag{3a}$$

$$\frac{\partial \rho v(x,0)}{\partial x}=-c'^2\frac{\partial \rho}{\partial x}-f\frac{\rho v^2}{2D} \tag{3b}$$

Boundary conditions are known to be depending on valves operational time and locations couple with compressor supply and pressure regulator. In these research valves are place in upstream and downstream of pipe. For a closure downstream valve and while up stream valve remain open boundary conditions are

$$P(0,t)=P_0(known\ function) \tag{4a}$$

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$$\frac{\partial(\rho u)}{\partial x} = (\rho u)_0 \, (known\, function) \tag{4b}$$

$$\frac{\partial p(L,t)}{\partial x} = 0 \tag{4c}$$

$$\rho(L,t) = \rho_L(t) \tag{4d}$$

For homogenous mixture of hydrogen natural, gas mixture density will be based on fluid mass ratio. We take into account hydrogen fluid mass ratio in this research to determining mixture density.

$$\varphi = m_h / (m_h + m_g) \tag{5}$$

where m_g , m_h are natural gas mass, hydrogen mass present and φ mass ratio (Elaoud et al., 2010, Elaoud and Hadj-Taı¨eb, 2008).

Considering a polytropic process of gas flow then the density of each gas present will be
The mixture density is then given by

$$\rho = \left[\frac{\phi}{\rho_h} + \frac{1-\phi}{\rho_g}\right]^{-1} = \left[\frac{\phi}{\rho_{h0}}\left(\frac{p}{p_0}\right)^{\frac{1}{nh}} + \frac{(1-\phi)}{\rho_{g0}}\left(\frac{p}{p_0}\right)^{\frac{1}{ng}}\right]^{-1} \tag{6}$$

(Elaoud and Hadj-Taı¨eb, 2008)

The celerity pressure of compressible gas fluid is define as

$$c' = \left(\frac{d\rho}{dp}\right)^{\frac{1}{2}} \tag{7}$$

Differentiating 6 and substituting into 7 we have

$$c' = \left[\frac{\phi}{\rho_{h0}}\left(\frac{p}{p_0}\right)^{\frac{1}{nh}} + \frac{(1-\phi)}{\rho_{g0}}\left(\frac{p}{p_0}\right)^{\frac{1}{ng}}\right] \times \left[\frac{1}{p}\left[\frac{\phi}{nh\rho_{h0}}\left(\frac{p}{p_0}\right)^{\frac{1}{nh}} + \frac{(1-\phi)}{ng\rho_{g0}}\left(\frac{p}{p_0}\right)^{\frac{1}{ng}}\right]\right]^{-\frac{1}{2}} \tag{8}$$

(Elaoud and Hadj-Taı¨eb, 2008)

Every flow is initially at steady state condition therefore the transient flow of HCNG is assumed
Finite Element Form of governing equation

Before partial differential equation toolbox can be used, both the governing equation and boundary conditions have to be rewritten in finite element format.

Rewriting the equations in the form

$$U_t - div(c \otimes grad(U)) + aU = F \tag{9}$$

Where

$$U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}; \; f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}; a = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}; \quad c = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}; d = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$$

Writing equations 1 and 2 in matrix form

$$\frac{\partial Q}{\partial t} + \frac{\partial E(Q)}{\partial x} - H(Q) = 0 \tag{10}$$

$$where \;\; Q = \begin{pmatrix} \rho \\ \rho u \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \tag{11}$$

$$E(Q) = \begin{pmatrix} q_2 \\ \dfrac{q_2^2}{q_1} + c'^2 q_1 \end{pmatrix} \tag{12}$$

$$H(Q) = \begin{pmatrix} 0 \\ -\dfrac{q_2 f\,|q_2|}{2q_1 D} \end{pmatrix} \tag{13}$$

$$d_{11}\frac{\partial q_1}{\partial t} + d_{12}\frac{\partial q_2}{\partial t} - c_{11}\frac{\partial q_1}{\partial x} - c_{12}\frac{\partial q_2}{\partial x} + a_{11}q_1 + a_{12}q_2 = f_1 \tag{14}$$

$$d_{21}\frac{\partial q_1}{\partial t} + d_{22}\frac{\partial q_2}{\partial t} - c_{21}\frac{\partial q_1}{\partial x} - c_{22}\frac{\partial q_2}{\partial x} + a_{21}q_1 + a_{22}q_2 = f_2 \tag{15}$$

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \end{bmatrix} - \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \frac{\partial q_1}{\partial x} \\ \frac{\partial q_2}{\partial x} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{16}$$

becomes

$$dQ_t - \nabla \cdot (c \otimes \nabla) + aQ = F \tag{17}$$

$$\frac{\partial q_1}{\partial t} + 0\frac{\partial q_2}{\partial t} - \frac{\partial q_1}{\partial x} - (-1)\frac{\partial q_2}{\partial x} + 0q_1 + 0q_2 = 0 \tag{18}$$

$$\frac{\partial q_1}{\partial t} + \frac{\partial q_2}{\partial t} - (-c'^2)\frac{\partial q_1}{\partial x} - \left(2\frac{q_2}{q_1}\right)\frac{\partial q_2}{\partial x} + 0q_1 + \left(-\frac{f\,|q_2|}{2q_1}\right)q_2 = 0 \tag{19}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ -c'^2 & -2\frac{q_2}{q_1} \end{bmatrix}\begin{bmatrix} \frac{\partial q_1}{\partial x} \\ \frac{\partial q_2}{\partial x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{f|q_2|}{2q_1} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} Q_t - \nabla \cdot \left(\begin{bmatrix} 0 & -1 \\ -c^2 & -2\frac{q_2}{q_1} \end{bmatrix} \otimes \nabla \right) + \begin{bmatrix} 0 & 0 \\ 0 & \frac{f|q_2|}{2q_1} \end{bmatrix} Q = \begin{bmatrix} 0 \\ 0 \end{bmatrix} F \tag{21}$$

(21) is nonlinear and PDEtool does NOT support nonlinear hyperbolic and parabolic PDE`s(Kaist, 2012, Yang et al., 2005)

Therefore a process of linearization will be employed, such that (21) becomes

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$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} Q_t - \nabla \cdot \left(\begin{bmatrix} 0 & -1 \\ -c'^2 & -2\frac{q_2'}{q_1'} \end{bmatrix} \otimes \nabla \right) + \begin{bmatrix} 0 & 0 \\ 0 & \frac{f|q_2'|}{2q_1'} \end{bmatrix} Q = \begin{bmatrix} 0 \\ 0 \end{bmatrix} F \qquad (22)$$

where q_1' and q_2' are steady state solution (1) and (2)

Boundary Condition will be written in Finite Element Form

Therefore

$c_{11} = 0,\ c_{12} = u_2,\ c_{21} = -C^2 u_1,\ c_{22} = uu_2$

$a_{11} = 0,\ a_{12} = 0,\ a_{21} = 0,\ a_{22} = u_x - \frac{uf}{2d}$

$f_1 = 0,\ f_2 = 0,\ d_{11} = 1,\ d_{12} = 0,\ d_{21} = 0,\ d_{22} = 1$

Used GUI to obtain the boundary matrix and the geometry composition of the define problem on the given domain, of the pipe dimension of d=0.41 with length 71.00m. This is used in the command syntax that solves nonlinear partial differential equations.

The concept of Newton iteration with Armijo-Goldstein line algorithm is used by the command. Using GUI to obtain the boundary matrix and the mesh parameters hence exported to MATLAB M-file command. The nonlinear parts are handle using pdeintrp command. To validate the method the problems present on B.G Agaie (2014) was solved using considered

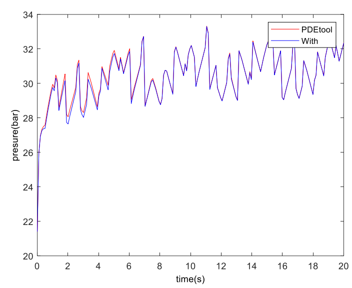

Figure 1 Validation of Method used on Pressure drop at outlet of the pipeline

From the result in figure 1 there is a clear agreement previous result, when outlet pressure drop is consider using the present method and reduce order modelling with static correction method.

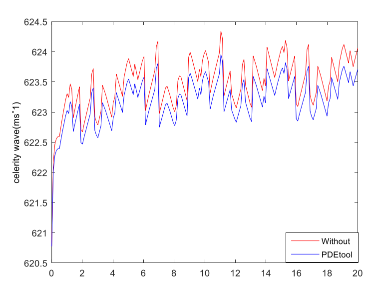

Figure 2 Celerity wave variation validation with ROM without Static Correction

The present method was also compared with ROM without static correction on flow celerity wave with a clear agreement as shown in figure 2

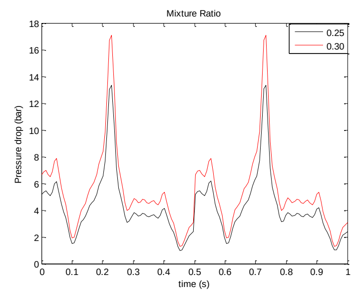

Figure 3: Effect of Hydrogen inclusion into natural gas on pressure drop.

Constantly of pressure drop can also be effected by increase on hydrogen present throughout the simulation process. This was observed from figure 3 only at time 0.54 second of the entire process. The highest pressure drop is observe at 0.25 and 0.75 seconds that is the first and last quarter of the process. The figure is observed to be a sinusoidal

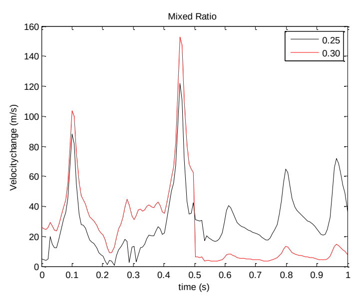

Figure 4: Velocity change for amount of hydrogen present on gas mixture

From the beginning of the simulation, velocity change due to hydrogen present is higher for increase on hydrogen present this is as a result of low gas density and burning ration that is established on hydrogen gas. It later stabilized for sometimes during the simulation process.

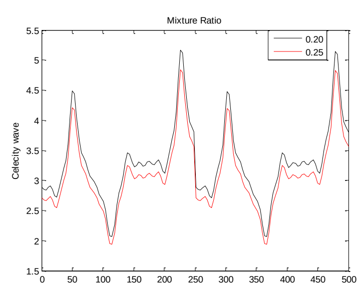

Figure 5: Celerity wave at different hydrogen present on gas mixture

The effect hydrogen on celerity wave is constant throughout the simulation process. At mid-point of the pipeline the celerity is equal for different in hydrogen percentage present during the flow.

Problem 2

Effect of body force on gas flow in pipeline

In this problem the momentum equation varies from equation (2) to take carry of the incline angle and becomes

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2+c^2\rho)}{\partial x}+\frac{f\rho u|u|}{2d}-\rho g\sin\theta=0 \qquad (23)$$

The process follows to obtain the result is as above equations 17 which becomes

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}Q_t-\nabla\cdot\left(\begin{bmatrix}0 & -1\\ -c^2 & -2\frac{q_2}{q_1}\end{bmatrix}\otimes\nabla\right)+\begin{bmatrix}0 & 0\\ 0 & \frac{f|q_2|}{2q_1}-\rho g\sin\theta\end{bmatrix}Q=\begin{bmatrix}0\\ 0\end{bmatrix}F \qquad (24)$$

Linearizing (24) asin (17) we have

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}Q_t-\nabla\cdot\left(\begin{bmatrix}0 & -1\\ -c'^2 & -2\frac{q_2'}{q_1'}\end{bmatrix}\otimes\nabla\right)+\begin{bmatrix}0 & 0\\ 0 & \frac{f|q_2'|}{2q_1'}-\rho g\sin\theta\end{bmatrix}Q=\begin{bmatrix}0\\ 0\end{bmatrix}F \qquad (25)$$

with q_1' and q_2' are steady state solution obtain from(1) and (2)

4. Results and Discussions

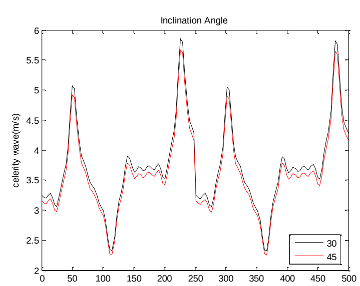

Figure 6: Effect of body force celerity wave of gas mixture

The effect of body force can be observed in the celerity wave of the pipeline system. Figure 4 the inclined angle is varies by a different of just 5^0 but the clearly show variation in the velocity wave.

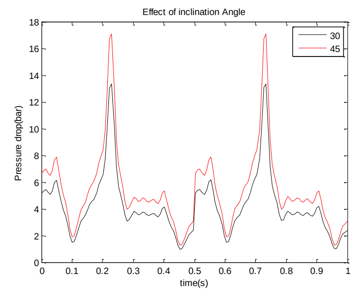

Figure 7: Effect of inclination angle on gas mixture pressure drop

There is a clear similarity on the pressure drop effect when compared to that of velocity wave as presented in figure 5 when the same inclination angles are varies.

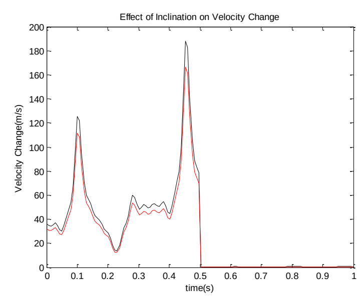

Figure 8: Effect of body force on gas mixture velocity change

From figure 8 the inclined affect the velocity change in each nodal point and converge at mid-point of the pipe length.

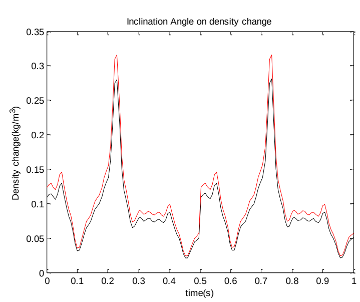

Figure 9: Effect of body force on gas mixture density change

From figure 9 the density change during flow at nodal point has no significant change on graph due to different in inclined angles

5. Conclusion and Recommendation

The method is clearly verified with the previous published result. The simulated results shows the accuracy on PDEtool with a significant agreement from previous result. The tool show to improve on accurate prediction of flow parameters. It is therefore recommended for simulation of flow related problem which can also be extended to system of three PDE that is when energy equation is involved to makes the problem a real life problem.

References

Agaie, B. G (2014). Numerical Computation of Transient Hydrogen Natural Gas Mixture Modelling (PhD) UniversitiTeknologi Malaysia

Agaie, B. G. and Mohammed Garba (2020). The Hydrogen Natural Gas Mixture Transient Flow at Different Celerity Wave

Abbaspour, M., Chapman, K. S. & Glasgow, L. A. 2010. Transient modeling of non-isothermal, dispersed two-phase flow in natural gas pipelines. *Applied Mathematical Modelling,* 34**,** 12.

Baufume, S., Hake, J. R.-F., Linssen, J. & Markewitz, P. 2011. Carbon capture and storage: A possible bridge to a future hydrogen infrastructure for Germany. *International journal o f hydrogen energy,* 36**,** 13.

Behbahani-Nejad, M. & Bagheri, A. 2010. The accuracy and efficiency of a MATLAB-Simulink library for transient flow simulation of gas pipelines and networks. *Journal of Petroleum Science and Engineering,* 70**,** 9.

Behbahani-Nejad, M. & Shekari, Y. 2010. The accuracy and efficiency of a reduced-order model for transient flow analysis in gas pipelines. *Journal of Petroleum Science and Engineering,* 73**,** 6.

Chaczykowski, M. 2010. Transient flow in natural gas pipeline the effect of pipeline thermal model. *Applied Mathematical Modelling,* 34**,** 16.

Chaudhry, M. H. 2008. *Open-Channel Flow,* Springer.

Cheng R, Littlejohn D, Strakey P & T, S. 2009. Laboratory investigation of a low-swiril injection H_2 and CH_4 at gas turbine conditions. . Proceedings of the combustion Instituted.

Elaoud, S. & Hadj-Taı¨Eb, E. 2008. Transient flow in pipelines of high-pressure hydrogen–natural gas mixtures. *International journal of hydrogen energy,* 33**,** 8.

Elaoud, S., Hadj-Taïeb, L. & Hadj-Taïeb, E. 2010. Leak detection of hydrogen-natural gas mixtures in pipes using the characteristics method of specified time intervals. *Journal of Loss Prevention in the Process Industries,* 23**,** 637-645.

Haeseldonckx, D. & D'haeseleer, W. 2011. Concrete transition issues towards a fully-fledged use of hydrogen as an energy carrier: Methodology and modelling. *International journal o f hydrogen energy,* 36.

Hoffman, A. K. & Chaiang, S. T. 2000. *Computational Fluid Dynamics For Engineers* Wichita Kansas, Engineering Education System.

Jang, S. P., Cho, C. Y., Nam, J. H., Lim, S.-H., Shin, D. & Chung, T.-Y. 2010. Numerical study on leakage detection and location in a simple gas pipeline branch using an array of pressure sensors. *Journal of Mechanical Science and Technology,* 24(4)**,** 7.

Jun, S., Park, K.-H., Kang, H.-M., Lee, D.-H. & Cho, M. 2010. Reduced order model of three-dimensional Euler equations using proper orthogonal decomposition basis. 24**,** 8.

Kaist, J. C. 2012. *Differential Equations in MATLAB 7* [Online]. www.mathnet.or.kr/real/2012/01/ChungJaywan1(0110).pdf. [Accessed 6th June 2012 2012].

Kessal, M. 2000. Simplified numerical simulation of transients in gas networks. *Institution of Chemical Engineers,* 78**,** 6.

Kiuchi, T. 1994. An implicit method for transient gas flows in pipe networks. *International Journal of Heat and Fluid Flow,* 15**,** 6.

Mohammadi, A., Shojaei, P. & Arabi, A. 2012. Determining the Optimum Route of Gas Pipeline by Environment Quantitative Approach Case Study: Fars Gas Company. *International Journal of Business & Management Tomorrow,* 2.

Osiadacz, A. J. & Chaczykowski, M. 1998. comparison of isothermal and non-isothermal transient models. *PSIG Annual Meeting* Denver, Colorado: Pipeline Simulation Interest Group.

Rahimpour, M. & Tavakoli, A. 2011. Multi-grid Beam and Warming scheme for the simulation of unsteady flow in an open channel. *Water Research,* 37**,** 8.

Romanowski, M. C. & Dowell, E. H. 1997. Reduced order Euler equation for unsteady aerodynamic flows. *In:* 34 (ed.) *Aerospace Science Meeting and Exhibit* Reno,NV: AIAA.

Stoianov, I., Nachman, L., Madden, S. & Tokmouline, T. 2007. PIPENETa wireless sensor network for pipeline monitoring. *Proceedings of the 6th international conference on Information processing in sensor networks.* Cambridge, Massachusetts, USA: ACM.

Tabkhi F, C, A.-P., I, P. & A, D. S. 2008. Mathematical frameworks of modelling and evaluation natural gas pipeline network under hydrogen injection. *international journal for hydrogen energy,* 4**,** 4.

Thorley, A. & Tiley, C. 1987. Unsteady and transient flow of compressible fluids in pipelinesda review of theoretical and some experimental studies. *International Journal of Heat and Fluid Flow,* 8**,** 13.

Uilhoorn, F. E. 2009. Dynamic behaviour of non-isothermal compressible natural gases mixed with hydrogen in pipelines. *International journal of hydrogen energy,* 34**,** 7.

Wang, G. 2011. The role of hydrogen cars in the economy of California. *Internation journal o f hydrogen energy,* 36**,** 9.

Winter, C.-J. 2009. Hydrogen energy d Abundant, efficient, clean: A debate over the energy-system-of-change. *Internation journal o f hydrogen energy* 34**,** 52.

Yang, C. & Ogden, J. 2007. Determining the lowest-cost hydrogen delivery mode. *International journal of hydrogen energy,* 32**,** 268-286.

Yang, W. Y., Cao, W., Chung, T.-S. & Morris, J. 2005. *Applied Numerical Methods Using MATLAB,* Canada, John Wiley & Sons, Inc., Hoboken, New Jersey.

Zhou, J. & Adewumi, M. A. 1996. Simulation of transient flow in natural gas pipelines *In:* 27TH (ed.) *Annual Meeting Pipeline Simulation Interest Group (PSIG).* Albuquerque New Mixico.