KASU JOURNAL OF MATHEMATICAL SCIENCES (KJMS) VOL. 3, ISSUE 1, JUNE 2022
ISSN 2734-3839 (Print), ISSN 2735-962X (E-Copy)
http://www.journal.kasu.edu.ng/index.php/kjms

KASU JOURNAL OF MATHEMATICAL SCIENCES

A Multi-Period Mixed Integer Programming (MIP) Model for a Metropolis Water Distribution

[1]Abdullahi N., [2]Awwalu H.B., [3]Dari S., [4]Yakubu M. I. and [5]Tahir U.

[1]Department of Mathematics, Nigerian Defence Academy, NDA Kaduna
nabdullahi@nda.edu.ng +2348035587641
[2]Department of Mathematics and Statistics, Hussaini Adamu Federal Polytechnic Kazaure
iyamibbg@gmail.com
[3]Department of Mathematical Sciences, Kaduna State University
sanisdari@yahoo.com
[4]Department of Mathematics, A. B. U. Zaria
ymariyat@yahoo.com
[5]Department of Mathematical Sciences, Kaduna State University
usmanmath309@yahoo.com

Abstract
Water production and distribution are expensive economic activities that affect the quality of life and the health of people in cities throughout the world. To facilitate smooth distribution of water through aqueducts and pipes in a growing and expanding city from production plants to service reservoirs, then to customers' zones (localities), the metropolis is divided into pressure zones that receive water concurrently. In this paper, a mixed integer programming formulation of water distribution, as multi-period (dynamic), two-echelon, and single-source, was analyzed as capacitated facility location problem by locating service reservoirs and assigning them to customers' zones. An optimum water schedule at minimum cost is obtained. The optimal solution suggested a total amount of ₦838,616,335, in respect of annual maintenance and pumping cost of water distribution of the board. This gives a saving of ₦117,058284 which is about 12.25% of improvement over the status quo.

Keywords: Mixed integer programming, facilities, Water, distributions, AMPL

1. Introduction

Kaduna town and environs is situated at 10.52^0 North latitude, 7.44^0 East longitude and 614 meters elevation above sea level (Figure 1). The town has a combined projected population of about 1,688,490 and 1,941,323 for year 2010 and year 2015 respectively KSWB (2006). This paper deals with a multi-period two echelon single-commodity capacitated facility location problem. The objective is to minimize the maintenance and pumping costs during the planning horizon, defined as the net total expenditure on energy consumptions, rehabilitations and maintenance of pipe network, service vehicles, personnel costs and so on. The planning horizon consists of a set of time periods, T. At the beginning of the first time period there exists a set of locations for the plants P, and service reservoirs, R. Water is being produced (on daily basis) at three operational water treatment plants, called (*major facilities*) with known designed capacities that satisfy the water need of the populace within the metropolis. The major facilities are: (**1**). Kaduna North Old Water Works (KN-OWW), commissioned in 1971, denoted by P_1 located at Malali with original designed capacity 90,000,000 liters per day equivalent to **90,000m^3/day**. (**2**). Kaduna North New Water Works (KN-WW), commissioned in 1987, denoted by P_2 located also at Malali, with original designed capacity of 150 mld, equivalent to **150,000m^3/day**. (**3**). Kaduna South, Water Works (KS-WW), commissioned in 1927, denoted by P_3 located at river Kaduna bank, near Railway station, with original designed capacity of 27mld, equivalent to **27,000m^3/day**, KSWB (2006). These three major facilities have combined original designed capacities of **267,000 m^3/day**. The quantity of water, produced at these major facilities, is being pumped via eleven (11) service reservoirs, called (*minor facilities*) with different designed capacities, to forty seven (47) localities called (*demand centers/zones*), within the metropolis. The demand for water at each demand center is known priori. The supply levels at the major facilities are subject to production capacities per day, also the supply levels at the minor facilities are bounded by the projected water demand of the localities and the reservoirs capacities. The demand of water by all the demand centers/zones cannot be splinted i.e. each demand center/zone is served by one minor facility only (single sourcing).

Now, the problem is to determine how to distribute the water between the open facilities (*major & minor*) and localities (*demand centers*) such that the sum of the fixed costs of maintaining the facilities and pumping (marginal) costs is minimized and such that all demand centers are served, and all supply levels (capacity restrictions) are satisfied. Therefore this is a flow problem. This work considered time – varying customers' demand in a multi – period decision horizon. Single source constraint (indivisible demand/ total assignment) formulation is often used in applications like ours in which the assignment cost is not proportional to demand, but represents "a fixed charge" cost in establishing link between the customer and the facility (in this case, reservoirs). For example, pipe network in water distribution, the assignment cost reflects the cost of laying a pipe connection (or in this case maintaining them) between customers' zone and service reservoirs, and does not depend on the volume of water (water demand) needed by the customers' zone, Geoffrion and Graves (1974). Changing customers' demand conditions, both temporal (relating to measured time) and spatial (relating to occupying space), necessitates making flow decisions accordingly so that customers are served with improved efficiency and cost effectiveness. These changes can be caused by several phenomena that are common in our today's economy. For example, system break down and sometime natural disaster, such as flooding, fire outbreak due chlorine explosion at treatment plants. Human factors, such as negligence of duty, cable theft. Other factors are: intermittent power supply, inadequate supply of diesel for stand by electric generators, all these leading to changes in both flow of supply to meet demand and the production capacities of treatment plants and service reservoirs. With the increase in urban water demand, new methods of efficient distribution of water for local use and in industries are required. The challenge of planning, designing, and managing urban water system is more and more difficult as different systems and technologies in water industry involve a number of disciplines which are needed for better planning. These disciplines includes: mechanics, electricity, communications, computer hardware and software, control methods, and mathematics (Izquierdo *et al.*, 2004). The common capacitated location problem consists of a set of customers with known demands which has to be assigned to a set of facilities/plants with enough capacities in order to meet the customers' demand. Most of the objective function contains costs of assigning customer to facilities/plants or in some instances the marginal cost of transporting/shipping a unit quantity of demand and facilities fixed opening/operating costs (fixed charge).

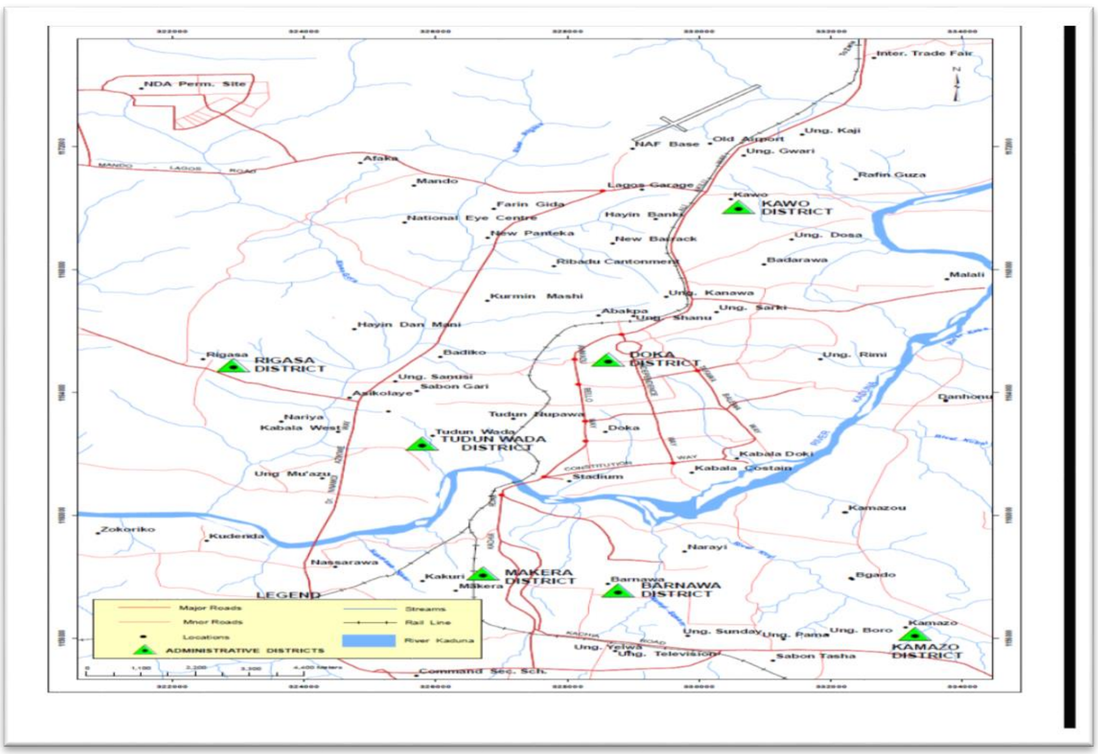

Fig. 1 Map of Kaduna Metropolis

Generally, this model considered that a customer can be assigned to one or more facilities/plants. In this situation, after fixing a subset of plants open, the optimal assignment of customers to open plants corresponds to the optimal solution of a classical transportation problem. A complicated version of this problem stem from the case when customers' demand must be satisfied by exactly one plant (total assignment or single sourcing)

2. Related work

There is enormous literature on two-echelon capacitated and uncapacitated location-distribution problems. Particularly, on the solutions strategies, several authors study relaxations approaches on the choice of formulations of these problems. For example, in an arc based (Network or sometime known as multi-commodity) formulation: Marin (2007), Pirkul and Jayaraman (1996, 1998), or a path based formulation, Barros and Labbe (1994), Gao and Robinson (1992) and Ro and Tcha (1984), or a comparisons of relaxations on alternative formulations, Bloemhof-Ruwaard *et al*. (1994, 1996), Marin and Pelegrin (1999) and Litvinchev *et al*. (2010). In a series of works by Dias *et al*. (2006, 2007 and 2008), a dynamic multi commodity location problems were considered and a primal-dual heuristic was developed that was able to find primal feasible solutions for the models. The first work of Dias *et al*. (2006), was a linear mixed integer model for the dynamic location problems (DLP) with discrete expansion and reduction sizes of available capacities. The model develop here, considered fixed charge costs for opening the first facility at any location, plus additional fixed costs for every opened facility in a location with already existing facilities. The model allowed the possibility of expanding or reducing the maximum available capacity, and also to open, close and reopen any facility at any location more than once during the planning horizon. In the second work, Dias *et al*. (2007), three capacitated DLP with opening and reopening of facilities are formulated. The first problem addresses the maximum capacity restrictions, while in the second problem; both maximum and minimum capacity restrictions were considered. The last problem take care of the situation where facility can be open (reopen) with its maximum capacity been decreases as customers are assigned to it during its operating periods. The third work, Dias *et al* (2008), several multi-level capacitated and uncapacitated DLP were formulated as MIP. The models here considered the possibility of opening, closing and reopening more than once the facilities during the planning horizon. The models may include both upper and lower limits on the used capacity of each facility and takes care of the situation where there is no flow conservation in the intermediate facilities. Hinojosa *et al* (2000), studied multi-period two-echelon multi-commodity CPLP. The studies considered when one desire to establish facilities at two different distribution levels by selecting the time periods. The model minimizes the total cost for meeting demands for all the products specified over the planning horizon at various customers' locations while, satisfying the capacity requirements of the facilities. A lagrangian relaxation was proposed together with a heuristic procedure that constructs feasible solutions of the original problem from the solutions at the lower bounds obtained by the relaxed problems. In ~~a~~ similar works, Canel *et al*. (2001) and Behmardi and Lee (2008), studied the same problem as in the case of Hinojosa et al (2000), but, in the former case a two parts algorithm that solve the problem was proposed. In the first part, branch and bound (B&B) was used to generate a list of solutions for each period and in the second part, dynamic programming was used to find an optimal sequence of conformation over the multi period planning horizon. Delta and Omega bounds (see Khumawala (1972), Akinc and Khumawala (1977), and Van Roy (1986)) were used extensively to effectively reduce the total number of transshipment sub-problems needed to be solved. But, Behmardi and Lee (2008) propose a model that allowed for change in the location and capacity of facilities over a time horizon to meet customers' demand. The authors recognize the fact that relocating facilities or changing the capacity takes significant time and resources, which would leads to unavailability of facilities or their full capacity. The proposed model takes care of this unavailability during the relocation or change of the capacity. In another related work on the multi-period dynamic CPLP

(MDCPLP) by Torres-Soto and Uster (2011), two multi-period dynamic-demand CPLP was considered (the problem is multi product). In the first problem, the facilities were allowed to be relocated in each period, whereas in the second problem, facilities are kept at fixed locations which are determine at the beginning of the planning horizon. Three solution strategies were considered, these include a lagrangian relaxation, Benders decomposition and ε-optimal Benders decomposition algorithms for the first model and a Benders decomposition algorithm for the second problem. Analysis was carried out on some generated instances to test the performance of the algorithms. Xu *et al.* (2012) attempts to address the urban water shortage problems by dedeveloping an effective decision-support tools for supporting water-supply schemes'under multiple uncertainties. A combined stochastic chance-constrained programming (SCCP) and interval linear programming (ILP) was use in an integrated general optimization frame work called interval-parameter stochastic chance-constrained programming (IPSCCP). The frame work IPSCCP, is capable in dealing with uncertainties when expressed as both discrete intervals and probability distributions. It can also be used to analyze the reliability of satisfying systems constraints. However, the method is inexact, it only help urban water managers to gain an in-depth insight into the tradeoffs between systems cost and reliability of constraints satisfaction. In another article by Ezenwaji *et al.* (2014), a linear programming approach for optimal allocation of public water supply to the urban sectors of Enugu, Nigeria was carried out. The approach is to find optimal allocation of public water using linear programming to about four public sectors that include residential, commercial, public institutions and industries. Some factors are incorporated that limit the availability of some resources which help to determine the maximum amount of water a sector could have.

In general, there is sparse literature in the context of dynamic (multi-period) multi-level capacitated location problem, (Dias *et al.* (2006), Canel *et al.* (2001)). In fact the paper due to Hinojosa *et al.* (2000) was the first attempt to considered the composition of multi echelon and multi-period models of the capacitated plant/facility location problem, then followed by Canel *et al.* (2001). Unlike the static models which were widely studied by numerous authors; see for example, Aikens (1985), Andreas and Andreas (2005), Sahin and Sural (2007), Revelle *et al.* (2008), Melo *et al*. (2009), Alireza and Reza (2012) and Reza *et al.* (2014) for a bibliography and reviews on these problems. The single source CPL problems were studied by numerous authors, such as: Klincewicz and Luss (1986), Sridharan (1993), Holmberg *et al.* (1999), and Cortinhal and Captivo (2003). On the other hand the two-level CPLPSS was investigated by Geoffrion and Graves (1974), Tragantalernsak *et al.* (1997, 2000), Klose (1999), Ahuja *et al.* (2004) and Rainwater *et al.* (2011) among many others. Klincewicz and Luss (1986) and Sridharan (1993), described lagrangian relaxation heuristic algorithm for CPLPSS. The former, dualized the capacity constraints in a lagrangian fashion, the result of which is an uncapacitated (Simple) facility location problem as a subproblem. The subprolem was solved using the dual ascent algorithm, an add heuristic compliment the relaxations which was used to obtained an initial feasible solution. Also a final adjustment heuristic based on cost differential was used to improve the solution at the end of the lagrangian phase. In what seem a computational comparison between an exact algorithm and the commercial code CPLEX was proposed by Holmberg *et al.* (1999). A primal heuristic based on a repeated matching algorithm was incorporated into a lagrangian heuristic and a branch and bound method was used to solve the problem. In the work of Cortinhal and Captivo (2003), lower bounds for the problem were obtained through lagrangian relaxation. Upper bounds for the problem are also given by a lagrangian heuristics followed by search methods coupled by one tabu search metaheuristic. Geoffrion and Graves (1974), was first to study the general multicommodity (path baased formulation) version of the CPLPSS and solve it using Benders decomposition. Tragantalerngsak *et al.* (1997, 2000) studied pure 0 -1 flow path formulation of the problem (some time referred to as multicommodity formulation) with no fixed charge cost of setting a facility in the second echelon. Tragantalerngsak *et al.* (1997), proposed a subgradient optimization procedure to solve a lagrangian dual of the CPLPSS. Six heuristics based on the lagrangian relaxation were proposed for the mathematical model of the problem with total assignment (single sourcing) in both the first and second echelons. Their methods shows improvement on the duality gap of one third of the one obtained from trivial LP relaxations. While in Tragantalerngsak *et al.* (2000), a lagrangian relaxation-based B&B procedure was used to solve the problem the authors reported that the method provides smaller number of B&B trees with less CPU time when compared to LP-based B&B obtained from 0 – 1 integer package. Klose (1999, 2000) a linear programming LP based heuristic was proposed for this problem in the former. Various LP relaxations, valid inequalities and facets for this problem was used to refine iteratively the formulations (tighten). Feasible solutions were obtained from the current LP solution of two alternative formulations; that is the path-based (multicommodity) and arc based (network) assignment formulations. While in Klose (2000), the capacity constraint was relaxed in a lagrangian fashion and an uncapacitated facility location problem was obtained as a sub-problem. A best-bound was computed for this relaxation by means of a 'weighted' Dantzig-Wolfe decomposition approach. Some valid inequalities that were violated by the fractional primal solution of the dual master problem were added and dualized in order to strengthen the relaxation. In what looks like a generalization of CPLPSS; where customer's demand contains a "flexible demand". An approach that makes use of a highly efficient heuristic within a neighborhood search that addresses the two level structure of the optimization problem was presented. In this research effort, the problem has three important features or characteristics which distinguishes it from the rest of the literature in this area of study:

- **i.** First is the additional requirement to the multi-period (dynamic) two-echelon capacitated plant location problem to have a Single Source constraint,
- **ii.** Secondly, the objective function is linear in terms of the decision variables, unlike in the case of Canel *et al.* (2001), whose problem objective involve nonlinear terms.
- **iii.** Thirdly, the problem is a flow one, and the objective is to minimize the sum of fixed charge cost of maintaining/opening of the facilities and the pumping/shipping costs.

These issues are not the case in the literature cited so far. Most of the objective function of these studies involve locating/assigning a potential facility location from sets of locations.

This study was motivated in an application in urban water distribution problem, where each customer's zones water demand must be satisfied from a single source (i.e. reservoir).

Multi period two-level water distribution problem with single source constraint (MTLCPLPSS): MIP formulation of the model

The problem has the objective of minimizing the total cost for meeting demands specified over time at various customers' zones. This cost ~~is~~ comprises of the marginal cost of pumping water on both level plus the fixed cost of maintaining the facilities.

Assumptions and notations

Assumptions:

The problem assumes the following hypothesis:

i. There are no holding decision
ii. The set of customers' zones facility locations are finite and ~~considered~~ fixed
iii. The set of customers and facilities (Plants, reservoirs) are known priori
iv. The time horizon is chosen in accordance with the period lengths and the planning horizon
v. The set of customers' zones facilities do not change over time.

Notations:

Sets:

$I = \{1, \dots, m\}, i \in I$ be the set of Plants

$J = \{1, \dots, n\}, j \in J$ be the set of Customers' zones

$K = \{1, \dots, p\}, k \in K$ be the set of reservoirs

$T = \{1, \dots, t\}, t \in T$ be the set of Periods

Parameters:

$S_k^t =$ Capacity of reservoir k at time period t

$a_i^t =$ Capacity of plant i at time period t

$d_j^t =$ Demand of Customers' zone j at time period t

$c_{kj}^t =$ Unit cost of pumping water from reservoir k at time period t

$b_{ik}^t =$ Unit cost of pumping water from plant i to reservoir k at time period t

$g_k^t =$ Maintenance/Operating cost of reservoir k during time period t

$f_i^t =$ Maintenance/Operating cost of plant i during time period t

Decision variables

$$x_{kj}^t = \begin{cases} 1 \; if\ zone\ j\ is\ supplied\ from\ reservoir\ k\ at\ the\ begining\ time\ period\ t \\ 0 \qquad\qquad otherwise \end{cases}$$

$w_{ik}^t =$ Amount of water pumped from plant i to reservoir k at time period t

$$y_i^t = \begin{cases} 1 \;\; if\ plant\ i\ is\ operational\ at\ the\ begining\ of\ time\ period\ t \\ 0 \qquad\qquad otherwise \end{cases}$$

$$z_k^t = \begin{cases} 1 \;\; if\ reservoir\ k\ is\ operational\ at\ the\ begining\ of\ time\ period\ t \\ 0 \qquad\qquad otherwise \end{cases}$$

Now at the beginning of the first time period, there exist subsets $I_c\ and\ K_c$ of the whole set of plants and reservoirs respectively that can be closed at the end of any time period, $t \in \{1, \dots, T\}$, but once closed they cannot be reopened. Let $I_0\ and\ K_0$ be the set of feasible locations where there does not exist open plants and open reservoirs respectively. These can be opened at the beginning of any time period, but, once they are opened they would not be closed. Let $R_0^1\ and\ R_0^T$ be the minimum number of reservoirs that are open at the beginning of the first time period and at the end of last time period. Similarly, let $P_0^1 and\ P_0^T$ be the minimum number of plants that are open at the beginning of the first time period and at the end of last time period. Then the objective function is defined as:

(P) $$\min Z\,(x, w, y, z) = \sum_{t=1}^{T}\sum_{k=1}^{p}\sum_{i=1}^{m} b_{ik}^t\, w_{ik}^t + \sum_{t=1}^{T}\sum_{i=1}^{m} f_i^{\,t} y_i^t + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{kj}^t \left(x_{kj}^t d_j^t\right) + \sum_{t=1}^{T}\sum_{k=1}^{p} g_k^t\, z_k^t$$

CONSTRAINTS

Subject to $\sum_k x_{kj}^t = 1 \quad \forall\, j, t$ (1)

$\sum_j d_j^t\, x_{kj}^t \le s_k^t z_k^t \quad \forall\, k, t$ (2)

$\sum_i w_{ik}^t - \sum_j d_j^t\, x_{kj}^t \;= 0 \quad \forall\, k, t$ (3)

$\sum_k w_{ik}^t \le a_i^t y_i^t \quad \forall\, i, t$ (4a)

$\sum_i w_{ik}^t \le S_k Z_k \quad \forall k$ (4b)

$\sum_k z_k^t \ge R_0^1, \quad \sum_k z_k^t \ge R_0^T$ (5)

$\sum_i y_i^t \ge P_0^1 \quad , \quad \sum_i y_i^t \ge P_0^T$ (6)

$z_k^t \ge z_k^{t+1} \quad \forall\, k \in K_c, \forall\, t,\, z_k^t \le z_k^{t+1} \quad \forall\, k \in K_0, \forall\, t$ (7)

$$y_i^t \geq y_i^{t+1} \quad \forall\, i \in I_c,\ \forall\, t, \quad y_i^t \leq y_i^{t+1} \quad \forall\, i \in I_0, \forall\, t \tag{8}$$

$$w_{ik}^t \geq 0, \ \forall\, i, \forall\, k, \forall\, t \tag{9}$$

$$x_{kj}^t\,,\ y_i^t\,, z_k^t \in \{0,1\},\ \forall\, i, \forall\, j, \forall\, k, \forall\, t \tag{10}$$

Constraints (1) guarantee meeting the demand of each customer in each time period, while (3) is the flow conservation constraints. Which means the total amount of water leaving the plants must be equal to the amount entering the reservoirs. Constraints (2) ensure that the total flow leaving the reservoirs does not exceed its capacity in each time period. Similarly constraints (4) ensure that the total flow emanating from the plants does not exceed its capacity. Constraints (5) and (6) state the minimum number of reservoirs and plants that must be open at the first and last time period. (7) and (8) describes the sets $K = K_0 \cup K_c$ and $I = I_0 \cup I_c$. (9) and (10) defines the domain of the decision variables.

This is a formulation often used for multi period models, but the variables and constraints tend to grow exponentially when preprocessing phase is invoke; Manzini and Gebennini (2008), Dias *et al.* (2008), Canel *et al*. (2001), and Hinojosa *et al.* (2000).

Below, we presents an alternative and equivalent formulation to (**P**) denoted by ($\bar{\boldsymbol{P}}$), that remedied the exponential grow of formulation (**P**), latter will show that there is a natural surjection between the solution sets of (**P**) and that of ($\bar{\boldsymbol{P}}$).

Equivalent formulation to (P)

We introduced the following variables:

$\forall\, k \in K_0, \forall\, t,$ we defined

$$h_k^t = \begin{cases} 1 \ if\ reservoir\ k\ is\ operational\ at\ the\ begining\ of\ time\ period\ t \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

$\forall\, k \in K_c, \forall\, t < T - 1$, we defined

$$h_k^t = \begin{cases} 1 \ if\ existing\ reservoir\ k\ is\ remove\ at\ the\ end\ of\ time\ period\ t \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

$\forall\, k \in K_c,$ defined,

$$h_k^T = \begin{cases} 1 \ if\ reservoir\ k\ is\ operational\ during\ all\ the\ planning\ horizon \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

$\forall\, i \in I_0, \forall\, t,$ defined,

ISSN 2734-3839 (Print), ISSN 2735-962X (E-Copy)
http://www.journal.kasu.edu.ng/index.php/kjms

$$q_i^t = \begin{cases} 1 \; if\ plant\ i\ is\ operational\ at\ the\ begining\ of\ time\ period\ t \\ 0 \qquad otherwise \end{cases}$$

$\forall\, i \in I_c, \forall\, t < T-1$, defined,

$$q_i^t = \begin{cases} 1 \; if\ existing\ plant\ i\ is\ remove\ at\ the\ end\ of\ time\ period\ t \\ 0 \qquad otherwise \end{cases}$$

$\forall\, i \in I_c$, defined

$$q_i^T = \begin{cases} 1 \; if\ plant\ i\ is\ operational\ during\ all\ the\ planning\ horizon \\ 0 \qquad otherwise \end{cases}$$

We define the costs as follows:

1. $G_k^t = \sum_{r=t}^{T} g_k^r$ = Total operational cost of reservoir $k\ in\ time\ period\ t\ \forall t, \forall k \in K_0$
2. $G_k^t = \sum_{r=1}^{t} g_k^r$ = Total cost of removing reservoir k at the end of $time\ period\ t\ \forall t, \forall k \in K_c$
3. $F_i^t = \sum_{r=t}^{T} f_i^r$ = Total operational cost of plant $i\ in\ time\ period\ t\ \forall t, \forall i \in I_0$
4. $F_i^t = \sum_{r=1}^{t} f_i^r$ = Total cost of removing plant i at the end of $time\ period\ t\ \forall t, \forall i \in I_c$

Let $T_{kt} = \begin{cases} \{1,\dots,t\} & if\ k \in K_0 \\ \{t,\dots,T\} & if\ k \in K_c \end{cases}$ and $T_{it} = \begin{cases} \{1,\dots,t\} & if\ i \in I_0 \\ \{t,\dots,T\} & if\ i \in I_c \end{cases}$

Then (**P**) can be reformulated in terms of the variables, h_k^t, q_i^t and the costs G_k^t and F_i^t.

$$(\bar{\boldsymbol{P}}): minimize f(x,w,h,q) = \sum_{t=1}^{T}\sum_{k=1}^{p}\sum_{i=1}^{m} b_{ik}^t\, w_{ik}^t\, s_k^t + \sum_{t=1}^{T}\sum_{i=1}^{m} F_i^t\, q_i^t$$
$$\sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{kj}^t\, x_{kj}^t d_j^t + \sum_{t=1}^{T}\sum_{k=1}^{p} G_k^t\, h_k^t$$

Subject to $\sum_k x_{kj}^t = 1 \quad \forall\, j, \forall\, t$ (11)

$$\sum_{j=1}^{n} d_j^t\, x_{kj}^t \le s_k^t \sum_{r\in T_{kt}} h_k^r \;, \forall\, k, \forall\, t \qquad (12)$$

$$\sum_{i=1}^{m} s_k^t\, w_{ik}^t - \sum_{j=1}^{n} d_j^t\, x_{kj}^t = 0 \quad \forall\, k, \forall\, t \qquad (13)$$

$$\sum_{k=1}^{p} s_k^t\, w_{ik}^t \le a_i^t \sum_{r\in T_{it}} q_i^r \quad \forall\, i, \forall\, t \qquad (14)$$

$$\sum_{k\in K_0} h_k^1 + \sum_{k\in K_c}\sum_{t=1}^{T} h_k^t \ge R_0^1 \qquad (15a)$$

$$\sum_{k\in K_c}\sum_{t=1}^{T} h_k^t + \sum_{k\in K_c} h_k^T \ge R_0^T \qquad (15b)$$

$$\sum_{i\in I_0} q_i^1 + \sum_{i\in I_c}\sum_{t=1}^{T} q_i^t \ge P_0^1 \qquad (16a)$$

ISSN 2734-3839 (Print), ISSN 2735-962X (E-Copy)
http://www.journal.kasu.edu.ng/index.php/kjms

$\sum_{i\in I_0}\sum_{t=1}^{T} q_i^t + \sum_{i\in I_c} q_i^T \geq P_0^T$ (16b)

$\sum_{t=1}^{T} h_k^t = 1 \ \forall\, k \in K_c$ (17a)

$\sum_{t=1}^{T} h_k^t \leq 1 \ \forall\, k \in K_0$ (17b)

$\sum_{t=1}^{T} q_i^t = 1 \ \forall\, i \in I_c$ (18a)

$\sum_{t=1}^{T} q_i^t \leq 1 \ \forall\, i \in I_0$ (18b)

$x_{kj}^t \in \{1,0\},\ w_{ik}^t \geq 0\ \forall\, k, \forall\, j, \forall\, i, \forall\, t$ (19)

$h_k^t, q_i^t \in \{1,0\}\ \forall\, k, \forall\, i, \forall\, t$ (20)

The following result prove that both (**P**) and ($\bar{\boldsymbol{P}}$) are equivalent in the sense that they provide the same set of solutions.

Proposition: $f(x,w,z,y) = g(\acute{x},\acute{w},h,q)\ \ iff\ (x,w,z,y)$ is a feasible solution to problem (**P**) and $(\acute{x},\acute{w},h,q)$ is a feasible solution of problem, ($\bar{\boldsymbol{P}}$)

Proof: $\Longrightarrow$ let (x,w,z,y) be a feasible solution of problem (**P**)

We define $(\acute{x},\acute{w},h,q)$ as: $\acute{x} = x,\quad \acute{w} = w$,

$h_k^1 = z_k^1,\ \forall k \in K_0 \quad h_k^t = z_k^t - z_k^{t-1}\ \forall\, t > 1$

$q_i^1 = y_i^1,\ \forall\, i \in I_0 \quad q_i^t = y_i^t - y_i^{t-1}\ \forall\, t > 1$

And $k \in K_c\ \ h_k^T = z_k^T\ , h_k^t = z_k^t - z_k^{t+1},\ \ \forall t < T$

$\forall i \in I_c,\ \ q_i^T = y_i^T,\ q_i^t = y_i^t - y_i^{t+1}\ \ \forall t < T$

By constraints (7) respectively (8) and (10) we obtain , $h_k^t = \{1,0\},\ \ \forall t, \forall k$

Also, $q_i^t = \{1,0\},\ \forall i, \forall t.$ Now by definition of the variables, $h\ and\ q$, we have $y_i^t = \sum_{r\in T_{it}} q_i^r$ and $z_k^t = \sum_{r\in T_{kt}} h_{kr}^r$

Therefore, since (x,w,z,y) is a feasible solution of (**P**)., by substituting $y,\ and\ z$ in the constraints of (**P**) we see that $(\acute{x},\acute{w},h,q)$ satisfy the constraints of ($\bar{\boldsymbol{P}}$). Hence $(\acute{x},\acute{w},h,q)$ is a feasible solution of ($\bar{\boldsymbol{P}}$).

On the other hand, by definition of $G_k^t\ and\ F_i^t$

$$\forall\, k \in K_0, g_k^t = \begin{cases} G_k^t - G_k^{t+1} & \forall t < T \\ G_k^T & if\ t = T \end{cases}$$

And $\forall\, k \in K_c, g_k^t = \begin{cases} G_k^t - G_k^{t-1} & \forall t > 1 \\ G_k^1 & if\ t = 1 \end{cases}$

$\forall\, i \in I_0, f_i^t = \begin{cases} F_i^t - F_i^{t+1} & \forall t < T \\ F_i^T & if\ t = T \end{cases}$

And $\forall\, i \in I_c,\ f_i^t = \begin{cases} F_i^t - F_i^{t-1} & \forall t > 1 \\ F_k^1 & if\ t = 1 \end{cases}$

We obtain:

$$\sum_{t=1}^{T} \sum_{k=1}^{p} g_k^t\, z_k^t = \sum_{k \in K_0} \{\sum_{t=1}^{T-1} (G_k^t - G_k^{t+1}) \sum_{r=1}^{t} h_k^r + G_r^T \sum_{r=1}^{T} h_k^r\} + \sum_{k \in K_c} \{G_k^1 \sum_{r=1}^{T} h_k^r + \sum_{t=2}^{T} (G_k^t - G_k^{t-1}) \sum_{r=1}^{T} h_k^r\}$$

$$= \sum_{k \in K_0} \{\sum_{t=1}^{T} G_k^t h_k^t\} + \sum_{k \in K_c} \{\sum_{t=1}^{T} G_k^t h_k^t\}$$

$$= \sum_{t=1}^{T} \sum_{k=1}^{m} G_k^t\, h_k^t \qquad (21)$$

Similarly;

$$\sum_{t=1}^{T} \sum_{k=1}^{p} f_i^t\, f_i^t = \sum_{i \in I_0} \{\sum_{t=1}^{T-1} (F_i^t - F_i^{t+1}) \sum_{r=1}^{t} q_i^r + F_i^T \sum_{r=1}^{T} q_i^r\} + \sum_{i \in I_c} \{F_i^1 \sum_{r=1}^{T} q_i^r + \sum_{t=2}^{T} (F_i^t - F_i^{t-1}) \sum_{r=1}^{T} q_i^r\}$$

$$= \sum_{i \in I_0} \{\sum_{t=1}^{T} F_i^t q_i^t\} + \sum_{i \in I_c} \{\sum_{t=1}^{T} F_i^t q_i^t\}$$

$$= \sum_{t=1}^{T} \sum_{i=1}^{m} F_i^t\, q_i^t \qquad (22)$$

Therefore, $f(x, w, z, y) = g(\acute{x}, \acute{w}, h, q)$

$\Leftarrow$, conversely, let $(\acute{x}, \acute{w}, h, q)$ be a feasible solution of problem $(\overline{\boldsymbol{P}})$,

We define (x, w, z, y), as $x = \acute{x}$, $y = \acute{y}$, $z_k^t = \sum_{r \in T_{kt}} h_k^r \quad \forall\, k, t,$

$y_i^t = \sum_{r \in T_{it}} q_i^r \quad \forall\, i, t.$

From constraints (17) on the reservoirs and respectively for the plants (constraints (18) and (20)) we will obtain $z_k^t = \{1{,}0\}\ \ \forall k, t\ and\ \ y_i^t = \{1{,}0\}\, \forall i, t$ respectively. Now by the definition of Z and constraints (6-17) we obtain

$z_k^1 = \sum_{t=1}^{T} h_k^t = 1 \ \ \forall\, k \in K_c, \forall t$ and by (6-20) we obtain $z_k^t \geq z_k^{t+1} \forall\, k \in K_c, \forall t$ and $z_k^t \leq z_k^{t+1} \forall\, k \in K_0, \forall t$, this implies the variable z, satisfied the constraints (8). Now substituting $h\ and\ q\ by\ z\ and\ y$, in the remainder constraints of $(\overline{\boldsymbol{P}})$ we see that $z\ and\ y$ satisfy the constraints of (**P**). Therefore, (x, w, z, y) is a feasible solution of (**P**) and by (21) and (22) we have, $= f(\acute{x}, \acute{w}, h, q) = g(x, w, z, y)$ □

Problem ($\bar{\boldsymbol{P}}$) is a mixed integer programming problem with embedded simple plant location problem (SPLP). But, it is well known that the SPLP is NP-hard. Krarup and Pruzzan (1983). What this mean is that, one cannot expect to solve exactly large sizes of problem ($\bar{\boldsymbol{P}}$) in polynomial time. Therefore, usually a heuristic method is used to solve ($\bar{\boldsymbol{P}}$). But, for the metropolis water distribution problem the constraint matrix are sparse. That is the underlying activities matrix is not dense but, sparse. A solution approach using the modeling capabilities of AMPL modeling language was used in transcribing the model from its algebraic closed form to AMPL codes and the commercial solver, CPLEX 12.5.6.00 was used to solve problem (**P**).

3. Result and discussion

The optimal water distribution schedule to localities within the Kaduna metropolis was achieved by the model described in the above section, which is a mixed integer programming formulation based on the structure of two-level capacitated facility location problem. Table 1 show the characteristic of the model. The AMPL codes for both the model and its data instances are too large to be displayed here due to limited space.

Table 1: Problem Characteristics for the dynamic model

S/NO	SIZE	NO. OF VARIABLES	NO. OF CONSTRAINTS	NO. OF NON ZEROS	INTEGRALITY GAP
1	3×11×47	237	260	497	0%

Table 2: First level optimal daily water distribution schedule (m^3/day)

Volume of water sent from treatment plants to service reservoirs (m^3/day)					
Treatment plants	S/NO	Reservoirs	Volume of water received per period		
			1	2	3
Malali Old Water Works	1	Lugard Hall	18215	18220	18225
	2	State House	7171	7166	7161
	3	Old NDA	38	38	38
	4	Old Airport	38	38	38
	5	Mando	38	38	38
Malali New Water Works	6	State House	1514	1519	1525
	7	Old NDA	1235	1235	1235
	8	Old Airport	735	736	737
	9	Mando Road	1211	1212	1212
	10	Tudun Wada	9664	9664	9667
	11	Rigasa	6541	6542	6542
	12	Kammazo	10179	10183	10187
	13	Unguwar Mu'azu	3003	3003	3004

	14	Kakuri/Makera	4249	4251	4252
	15	Barnawa	4139	4140	4140
Kaduna South Water Works	16	Unguwar Mu'azu	27	27	27
	17	Kakuri/Makera	7626	7626	7626
	18	Barnawa	27	27	27

Table 3: Second level optimal daily water distribution schedule (m^3/day)

Volume of water sent from service reservoirs to customers' zone (m^3/day)					
Service reservoirs	**S/NO**	**Customers' zones**	**Volume of water received per period**		
			1	**2**	**3**
Lugard Hall	1	Hayin Banki	1182	1183	1183
	2	Unguwar Sarki	465	465	466
	3	Unguwar Kanawa	695	695	695
	4	Unguwar Shanu	1318	1319	1319
	5	Abakpa	968	968	968
	6	Unguwar Rimi	3351	3352	3352
	7	Kabala Doki	1442	1443	1443
	8	Kabala Constain	862	862	863
	9	Doka	3427	3427	3428
	10	Mahuta	39	39	40
	11	Sabon Gari	4466	4466	4467
State House	12	Kawo	2653	2653	2653
	13	Unguwar Dosa	1579	1579	1580
	14	Badarawa	2631	2631	2631
	15	Malali	1822	1822	1822
Old NDA	16	Kurmin Mashi	1273	1273	1273
Old Airport	17	Unguwar Gwari	685	685	685
	18	Mai Gero & others	50	50	51
	19	Na Mai Gero	38	39	39
Mando Road	20	Afaka	1170	1170	1170
	21	Sabon Afaka	79	80	80
Tudun Wada	22	Tudun Nupawa	3158	3158	3159
	23	Badiko	1034	1034	1034
	24	Nariya	115	115	116
	25	Unguwar Sunusi	1524	1524	1524
	26	Tudun Wada	3833	3833	3834
Rigasa	27	Rigasa	6541	6542	6542
Kammazo	28	U/ Television	2277	2277	2277
	29	U/ Sunday	549	549	550

	30	Unguwar Yelwa	2023	2024	2024
	31	Sabon Tasha	2459	2459	2459
	32	NNPC Qtars	150	150	151
	33	Unguwar Boro	40	40	40
	34	Pan Tuta	112	112	112
	35	Narayi	2136	2137	2137
	36	Bayan Dutse	38	39	39
	37	Zarma Zarma	26	26	27
	38	Gonin Gora	306	306	306
	39	Kammazo	40	41	41
	40	Jankasa	23	23	23
U\Mu'azu Kabala West	41	Unguwar Mu'azu	3030	3030	3031
Kakuri Makera	42	Kakuri & Makera	6216	6217	6217
	43	Nasarawa & Tirkaniya	4941	4942	4942
	44	Kudendan I	27	27	27
	45	Kudendan II	34	34	34
	46	Unguwar Romi	657	657	658
Barnawa	47	Barnawa & Environs	4166	4167	4167

Table 4: Computational Results

S/NO	**SIZE**	$\boldsymbol{Z}(.)$	$\bar{\boldsymbol{Z}}(.)$	$\frac{\boldsymbol{Z}(.)-\bar{\boldsymbol{Z}}(.)}{\boldsymbol{Z}(.)} \times \mathbf{100}$
1	3×11×47	₦2,297,579	₦2,297,579	0%

where; $Z(.) and\ \bar{Z}(.)$, denote optimal integer and optimal relaxed (continuous) solutions of the problem.

Tables 2 and 3 give the optimal water distribution schedules for the hierarchical (level) structure of the distribution network. Table 2 gives the daily optimal water schedule for the first level, i.e. from the treatment plants to the distributions reservoirs in m^3/day. Table 3 gives the daily optimal water schedule for the second level, i.e. from the distributions reservoirs to customers' zones in m^3/day.

We can observe also that, the single sourcing has been put into effects, where every service reservoir received supply from one treatment plant only. In similar vein every customers' zones is service by only one service reservoir.

Table 4 give the optimal (minimum) cost of all the daily schedules in naira. The difference between the new (minimum) cost and the existing status quo is reasonable and within the threshold of economy of alternatives.

4. Conclusion and implementation:

The current system being run by the KSWB, in respect of annual maintenance and pumping cost of water distribution is calculated as **₦955,674,619** (Kaduna state water board, 2006, 2013A, 2013B). From the solutions of the model, the values obtained (Table 2) show an improvement over the status quo, on the annual maintenance and pumping costs. The optimal solution suggested that the total amount to be spent is **₦838,616,335.** This gives a saving of **₦117,058284** which is about **12.25%.**

Acknowledgement

We thanks IBM Academic initiative program for granting us academic license for CPLEX Optimization Studio for Academic Research 12.6.3.0. We are also grateful to AMPL Optimization for granting us AMPL for course license.

References

Ahuja, R.K., Orlin, J.B., Pallatino, S., Seaparra, M.P. and Scutella, M.G. (2004) "A multi-exchange heuristic for the single source capacitated facility location problem" *Management Science.* 50(6):749 – 760.

Aikens C.H., (1985) "Facility location models for distribution planning", *European Journal of Operational Research.* 22: 263 – 279.

Akinc, U., and Khumawala, B.M. (1977) "An efficient Branch and Bound algorithm for the capacitated warehouse location problem", *Management Science.* 23: 585-594.

Alireza, B. A. and Reza, Z.F (2012) "Facility location dynamics: An overview of classifications and applications ", *Computers and Industrial Engineering.* 62: 408 – 420.

Andreas, K. and Andreas, D. (2005) "Facility location models for Distribution system design", *European Journal of Operational Research.* 162: 4 – 29.

Barros, I and Labbe, M., (1994) "A general model for the uncapacitated facility and depot location problem", *Location science.* 2: 173-191.

Behmardi, B. and Lee, S.,(2008) "Dynamic Multi-commodity capacitated facility location problem in supply chain", In J. Fowler and S. Mason, (eds). *Proceedings of the 2008 Industrial Engineering Research Conference*

Bloemhof-Ruwaard, J.M., Solomon, N., and Van-Wassenhove, L.N., (1994) "On the coordination of products and by products flows in two level distribution networks: Model formulation and solution procedure", *European Journal of Operational Research. 79* (3):325-339.

Bloemhof-Ruwaard, J.M., Solomon, N., and Van-Wassenhove, L.N., (1996) "The Capacitated Distribution and waste Disposal Problem", *European Journal of Operational Research. 88*(3): 490-503.

Canel, C., Khumawala, B.M., Law, J. and Loh, A., (2001) "An Algorithm for the capacitated multi-commodity multi-period facility location problem", *Computers and Operations Research.* 28: 411 – 427.

Cortinhal, M.J. and Captivo, M.E. (2003) "Upper and Lower bounds for the single source capacitated location problem", ", *European Journal of Operational Research.* 151: 333-351.

Dias, J., Capativo, ME., and Climaco, J. (2006) "Capacitated dynamic location problems with opening, closure and re opening of facilities", *IMA Journal of Management Mathematics.* 17(4): 317-348.

Dias, J., Capativo, ME., and Climaco, J. (2007) "Efficient Primal-dual heuristic for a dynamic location problem", *Computers and Operations Research.* 34(6): 1800-1823.

Dias, J., Capativo, ME., and Climaco, J. (2008) "Dynamic multi-level capacitated and uncapacitated location problems: an approach using primal-dual heuristics", *Operational Research. An International Journal. 7* (3): 345 – 379.

Ezenwaji, E.E., Anyadike, R.N.C., and Igu, N.I. (2014) "Optimal allocation of public water to the urban sectors of Enugu, Nigeria: a linear programming approach", *Applied Water Science.* 4: 73-78.

Gao, L.L., and Robinson, E.P., (1992) "A dual based optimization procedure for the two echelon un capacitated facility location problem", *Naval Research logistic.* 39: 191-212.

Geoffrion A.M. and Graves G.W.,(1974) "Multicommodity distribution system design by Benders decomposition", *Management Science.* 20: 822-844.

Holmberg, K., Ronnqvist, M. and Yuan, D. (1999) "An exact algorithms for the capacitated facility loction problems with single source", ", *European Journal of Operational Research.* 113: 544 – 559.

Hinojosa, Y., Puerto, J. and Fernandez, F.R., (2000) "A multiperiod two-eechelon capacitated plant location problem", *European Journal of Operational Research.* 123: 271 – 291.

Izquierdo J., Perez R. and Iglesias P.L., (2004) "Mathematical Models and Methods in the Water industry", *Mathematical and Computer Modelling.* 39: 1353 – 1374.

Kaduna State Water Board (KSWB), (2006), National Urban Water Sector Reform Project Rehabilitation and Network Expansion of Kaduna Metropolitan Water Supply Scheme. Credit No.39240, Detailed Engineering Design Report Phase II, Volume I, Main Report, Nov. 2006

Kaduna State Water Board (KSWB), (2013A), Kaduna North Plant, (Old and New) Malali Budget proposal for 2013-2015

Kaduna State Water Board (KSWB), (2013B), Kaduna South Plant and water Works, Budget proposal for 2013-2015

Khumawala, B.M., (1972) "An efficient branch and bound algorithm for the warehouse location problem", *Management Science.* 18: 718-731.

Klincewicz, J.G. and Lucas, H. (1986) "A lagrangean Relaxations heuristic for capacitated facility location with single source constraints" *Journal of the Operational Research Society.* 37 (5): 495 – 500.

Klose, A. (1999) "An LP based heuristic for two-stage capacitated facility location problems", *Journal of the Operational Research Society.* 50: 157 – 166.

Marin A., and Pelegrin, B. (1999) "Applying Lagrangian relaxation to the resolution of two stage location problem", *Annals of OperationalmReserch.* 86: 179-198.

Marín, A., (2007) "Lower bounds for the two-stage uncapacitated facility location problem" *European Journal of Operational Research.* 179: 1126-1142.

Melo, T., Nickel, S., Saldanha da Gama, F. (2009) "Facility location and Supply chain Management-a review", *European Journal of Operational Research*. 196:401 – 412.

Litvinchev, I., Mata, M. and Rangel S.,(2010) "Calculating the best dual bound for problems with multiple lagrangean relaxations", *Journal of Computer and systems sciences International"*. 49(6): 915-922.

Pirkul, H. and Jayaraman, V. (1996) "Production transportation and distribution planning in a multi commodity tri-echelon system", *Transportation Science.* 30: 291 – 302.

Pirkul, H. and Jayaraman, V. (1998) "A multi-commodity multi plant capacitated facility location problem: formulation and efficient heuristic solution", *Computers and Operations Research.* 25:869- 878

Rainwater, C., Geunes, J. and Romeijn, H.E. (2011) "A facility neighborhood search heuristic for capacitated facility location with single source constraints and flexible demand" *SPRINGER Journal of heuristics* doi: 10.1007/s10732-011-9179-x

Reza, Z.F. Masoud, H. Behman, F. and Narges, K.Z. (2014) "Hierarchical facility location problem: models, classification, techniques and applications" *Computers and Industrial Engineering.* 68:104 – 117.

Revelle, C.S., Eiselt, H.A., Daskin, M.S., (2008) "A bibliography for some fundamental problem categories in discrete location science", *European Journal of Operational Research.* 184:817 – 848.

Ro, H.B., and Tcha, D.W. (1984) "A branch and bound algorithm for the two level uncapacitated facility location problem with some side constraints", *European Journal of Operational Research.* 18:349-358.

Sahin, G. and Eural, H. (2007) "A review of hierarchical facility location models", *Computers and Operations Research.* 34:2310 – 2331.

Sridharan, R. (1993) "A lagrangian heuristic for the capacitated plant location problem with single source constraints" ", *European Journal of Operational Research.* 66:305 – 312.

Torres-Soto, J. and Uster, H., (2012) "Dynamic-demand capacitated facility location problems with and without relocation", *International Journal of Production research* 49 (13) 3979 – 4005.

Tragantalerngsak, S. Holt, J. and Ronnquist, M., (1997) "Lagrangian heuristics for the two-echelon single source capacitated facility location problem", *European Journal of Operational Research.* 102: 611 – 625.

Tragantalerngsak, S. Holt, J. and Ronnquist, M., (2003) "an exact method for the two-echelon single source capacitated facility location problem", *European Journal of Operational Research.* 123: 473 – 489.

Van Roy, T.J.,(1986) "A cross decomposition algorithm for capacitated facility location problem", *Operations Research.* 34(1): 145-163.

Xu,Y., Huang, G.H., and Xu, T.Y., (2012) "Inexact Management Modeling for Urban Water Supply Systems", *Journal of Environmental informatics.* 20 (1): 34 – 43.

Appendix A: First Level Optimal Daily Water Distribution Schedule (m^3/day)

Volume of Water Sent from Treatment Plants to Service Reservoirs (m^3/day)					
Treatment Plants	**S/NO**	**RESERVOIRS**	**VOLUME OF WATER RECEIVED PER PERIOD**		
			1	**2**	**3**
Malali Old Water Works	1	Lugard Hall	18215	18220	18225
	2	State House	7171	7166	7161
	3	Old NDA	38	38	38
	4	Old Airport	38	38	38
	5	Mando	38	38	38
Malali New Water Works	6	State House	1514	1519	1525
	7	Old NDA	1235	1235	1235
	8	Old Airport	735	736	737
	9	Mando Road	1211	1212	1212
	10	Tudun Wada	9664	9664	9667
	11	Rigasa	6541	6542	6542
	12	Kammazo	10179	10183	10187
	13	Unguwar Mu'azu	3003	3003	3004
	14	Kakuri/Makera	4249	4251	4252
	15	Barnawa	4139	4140	4140
Kaduna South Water Works	16	Unguwar Mu'azu	27	27	27
	17	Kakuri/Makera	7626	7626	7626
	18	Barnawa	27	27	27

Appendix B: Second Level Optimal Daily Water Distribution Schedule (m^3/day)

Volume of Water Sent from Service Reservoirs to Customers' Zone (m^3/day)					
SERVICE RESERVOIRS	**S/NO**	**CUSTOMERS' ZONES**	**VOLUME OF WATER RECEIVED PER PERIOD**		
			1	**2**	**3**
Lugard Hall	1	Hayin Banki	1182	1183	1183
	2	Unguwar Sarki	465	465	466
	3	Unguwar Kanawa	695	695	695
	4	Unguwar Shanu	1318	1319	1319
	5	Abakpa	968	968	968
	6	Unguwar Rimi	3351	3352	3352
	7	Kabala Doki	1442	1443	1443
	8	Kabala Constain	862	862	863
	9	Doka	3427	3427	3428
	10	Mahuta	39	39	40
	11	Sabon Gari	4466	4466	4467
State House	12	Kawo	2653	2653	2653
	13	Unguwar Dosa	1579	1579	1580
	14	Badarawa	2631	2631	2631
	15	Malali	1822	1822	1822

Old NDA	16	Kurmin Mashi	1273	1273	1273
Old Airport	17	Unguwar Gwari	685	685	685
	18	Mai Gero & others	50	50	51
	19	Na Mai Gero	38	39	39
Mando Road	20	Afaka	1170	1170	1170
	21	Sabon Afaka	79	80	80
Tudun Wada	22	Tudun Nupawa	3158	3158	3159
	23	Badiko	1034	1034	1034
	24	Nariya	115	115	116
	25	Unguwar Sunusi	1524	1524	1524
	26	Tudun Wada	3833	3833	3834
Rigasa	27	Rigasa	6541	6542	6542
Kammazo	28	U/ Television	2277	2277	2277
	29	U/ Sunday	549	549	550
	30	Unguwar Yelwa	2023	2024	2024
	31	Sabon Tasha	2459	2459	2459
	32	NNPC Qtars	150	150	151
	33	Unguwar Boro	40	40	40
	34	Pan Tuta	112	112	112
	35	Narayi	2136	2137	2137
	36	Bayan Dutse	38	39	39
	37	Zarma Zarma	26	26	27
	38	Gonin Gora	306	306	306
	39	Kammazo	40	41	41
	40	Jankasa	23	23	23
U\Mu'azu Kabala West	41	Unguwar Mu'azu	3030	3030	3031
Kakuri Makera	42	Kakuri & Makera	6216	6217	6217
	43	Nasarawa & Tirkaniya	4941	4942	4942
	44	Kudendan I	27	27	27
	45	Kudendan II	34	34	34
	46	Unguwar Romi	657	657	658
Barnawa	47	Barnawa & Environs	4166	4167	4167