



A Single Step Block Numerical Integrator for Direct Solution of Initial Value Problems of Third Order Ordinary Differential Equations

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Abstract

This article discusses a single step block numerical integrator for direct solution of initial value problems of third order ordinary differential equations. The derivation of the method is achieved via interpolation and collocation where approximated power series is assigned to be interpolating polynomial. The third derivative of this polynomial is taken as the collocating equations. These interpolating and collocating equations are then put together in a matrix form in order to find the unknown variables which are substituted into the interpolating polynomial to give the continuous implicit scheme. The discrete schemes and its derivatives that form the block are derived by evaluating the continuous implicit scheme as well as its derivatives at the non-interpolating points. The method is found to be order six, zero stable, consistent and also satisfies conditions for convergence. The potency of the method is tested on some third order initial value problems and the outcome proved its effectiveness over current existing methods.

1. Introduction

The new numerical scheme is specifically developed to solving Initial value problems of the form

$$y''' = f(x, y(x), y'(x), y''(x))$$

$$y(x_0) = y_0, y'(x_0) = y'_0, y''(x_0) = y''_0 \quad (1)$$

This method overcomes the setbacks attached to reduction technique as it handles equation (1) directly without reducing it to system of first order ordinary differential equations (ODEs). Several researchers had worked on providing approximate solution to the equation (1). Olabode and Yusuph (2009) proposed a linear multi-step method for the direct solution of initial value problems. The new method which is a 3-step block method was P-stable, consistent and accurate. Olabode (2013) however presented a direct block multi-step method which generate the non-overlapping solution at selected grip points which was better than the conventional predictor-corrector (P-C) methods for solving third order initial value problems. In the work of Kuboye et al. (2020), a numerical method was developed using multi-step collocation in deriving the methods and the order of the method was found to be six and the numerical results proved the efficiency of the method over the existing methods. Ogunware and Omole (2020) in their work considered the direct solution by three-step irrational linear multi-step method using collocation and interpolation

techniques. A symmetric hybrid continuous linear multi-step method was developed by Kayode and Obarhua (2017) and it was found to be symmetric, consistent, zero-stable and of order six with low error constants. These scholars mentioned above came up with multi-step collocation approach where step-number $k > 1$ was used in developing their schemes. This paper proposes a single-step ($k = 1$) numerical method of order six for solving equation (1) directly. In deriving the method, approximated power series is used as interpolating polynomial while its third derivative is assigned to be the collocating equation.

2. Methodology

2.1 Derivation Of Block Method

Power series of the form:

$$y(x) = \sum_{j=0}^{k+7} a_j x^j \quad (2)$$

is used as approximate solution to (1), where $k = 1$. Equation (2) is differentiated three times to give:

$$y'''(x) = \sum_{j=0}^{k+7} j(j-1)(j-2)a_j x^{j-3} \quad (3)$$

Equation (2) is interpolated at $x = x_{n+i}, i = 0(\frac{1}{5})\frac{2}{5}$ and collocating (3) at $x = x_{n+T}, T = 0(\frac{1}{5})1$. This produces

$$\begin{aligned} \sum_{j=3}^{k+7} a_j x_{n+i}^j &= y_{n+i} \\ \sum_{j=3}^{k+7} j(j-1)(j-2)a_j x_{n+T}^{j-3} &= f_{n+T}. \end{aligned} \quad (4)$$

In finding the values of $a_j, j=0(1)7$ in (4). Gaussian elimination technique is applied which are then substituted into (2) to give a continuous implicit scheme of the form

$$\begin{aligned} &\alpha_0(t)y_n + \alpha_{\frac{1}{5}}(t)y_{n+\frac{1}{5}} + \alpha_{\frac{2}{5}}(t)y_{n+\frac{2}{5}} + \alpha_{\frac{3}{5}}(t)y_{n+\frac{3}{5}} \\ &= h^3[\beta_0(t)F_n + \beta_{\frac{1}{5}}(t)F_{n+\frac{1}{5}} + \beta_{\frac{2}{5}}(t)F_{n+\frac{2}{5}} + \beta_{\frac{3}{5}}(t)F_{n+\frac{3}{5}} + \beta_{\frac{4}{5}}(t)F_{n+\frac{4}{5}} + \beta_1(t)F_{n+1}] \end{aligned} \quad (5)$$

The first and second derivatives of (5) are given as follow

$$\begin{aligned} &\alpha'_0(t)y_n + \alpha'_{\frac{1}{5}}(t)y_{n+\frac{1}{5}} + \alpha'_{\frac{2}{5}}(t)y_{n+\frac{2}{5}} + \alpha'_{\frac{3}{5}}(t)y_{n+\frac{3}{5}} \\ &= h^3[\beta'_0(t)F_n + \beta'_{\frac{1}{5}}(t)F_{n+\frac{1}{5}} + \beta'_{\frac{2}{5}}(t)F_{n+\frac{2}{5}} + \beta'_{\frac{3}{5}}(t)F_{n+\frac{3}{5}} + \beta'_{\frac{4}{5}}(t)F_{n+\frac{4}{5}} + \beta'_1(t)F_{n+1}] \end{aligned} \quad (6)$$

$$\alpha''_0(t)y_n + \alpha''_{\frac{1}{5}}(t)y_{n+\frac{1}{5}} + \alpha''_{\frac{2}{5}}(t)y_{n+\frac{2}{5}} + \alpha''_{\frac{3}{5}}(t)y_{n+\frac{3}{5}}$$

$$= h^3 [\beta''_0(t)F_n + \beta''_{\frac{1}{5}}(t)F_{n+\frac{1}{5}} + \beta''_{\frac{2}{5}}(t)F_{n+\frac{2}{5}} + \beta''_{\frac{3}{5}}(t)F_{n+\frac{3}{5}} + \beta''_{\frac{4}{5}}(t)F_{n+\frac{4}{5}} + \beta''_1(t)F_{n+1}] \quad (7)$$

where $t = \frac{x-x_n}{h}$

$$\alpha_0 = \frac{(25t^2)}{2} - \frac{(15t)}{2} + 1$$

$$\alpha_{\frac{1}{5}} = -25t^2 + 10t$$

$$\alpha_{\frac{2}{5}} = \frac{(25t^2)}{2} - \frac{5t}{2}$$

$$\beta_0 = \frac{(625h^3t^8)}{+} - \frac{(125h^3t^7)}{336} - \frac{(425h^3t^6)}{576} + \frac{(25h^3t^5)}{32} - \frac{(137h^3t^4)}{288} + \frac{(h^3t^3)}{6} - \frac{(1243h^3t^2)}{40320} + \frac{(127h^3t)}{56000}$$

$$\beta_{\frac{1}{5}} = \frac{3125h^3t^8}{8064} - \frac{(125h^3t^7)}{72} + \frac{(1775h^3t^6)}{576} - \frac{(385h^3t^5)}{144} + \frac{(25h^3t^4)}{24} - \frac{(2057h^3t^2)}{22400} + \frac{(967h^3t)}{72000}$$

$$\beta_{\frac{2}{5}} = -\frac{(3125h^3t^8)}{4032} + \frac{(1625h^3t^7)}{504} - \frac{(1475h^3t^6)}{288} + \frac{(535h^3t^5)}{144} - \frac{(25h^3t^4)}{24} + \frac{(461h^3t^2)}{11200} - \frac{(1109h^3t)}{252000}$$

$$\beta_{\frac{3}{5}} = \frac{(3125h^3t^8)}{4032} - \frac{(125h^3t^7)}{42} + \frac{(1225h^3t^6)}{288} - \frac{(65h^3t^5)}{24} + \frac{(25h^3t^4)}{36} - \frac{(2693h^3t^2)}{100800} + \frac{(247h^3t)}{84000}$$

$$\beta_{\frac{4}{5}} = -\frac{(3125h^3t^8)}{8064} + \frac{(1375h^3t^7)}{1008} - \frac{(1025h^3t^6)}{576} + \frac{(305h^3t^5)}{288} - \frac{(25h^3t^4)}{96} + \frac{(131h^3t^2)}{13440} - \frac{(541h^3t)}{504000}$$

$$\beta_1 = \frac{(625h^3t^8)}{8064} - \frac{(125h^3t^7)}{504} + \frac{(175h^3t^6)}{576} - \frac{(25h^3t^5)}{144} + \frac{(h^3t^4)}{24} - \frac{(103h^3t^2)}{67200} + \frac{(17h^3t)}{100800}$$

$$\alpha'_0 = \frac{(12600000t - (3780000))}{(504000h)}$$

$$\alpha'_{\frac{1}{5}} = -\frac{(25200000t - 5040000)}{(504000h)}$$

$$\alpha'_{\frac{2}{5}} = \frac{(12600000t - 1260000)}{(504000h)}$$

$$\beta'_0 = \frac{(312500h^3t^7 - 1312500h^3t^6 + 2231250h^3t^5 - 1968750h^3t^4 + 959000h^3t^3 - 252000h^3t^2 + 31075h^3t - 1143h^3)}{(504000h)}$$

$$\beta'_{\frac{1}{5}} = \frac{(1562500h^3t^7 - 6125000h^3t^6 + 9318750h^3t^5 - 6737500h^3t^4 + 2100000h^3t^3 - 92565h^3t^2 + 6769h^3)}{(504000h)}$$

$$\beta'_{\frac{2}{5}} = - \frac{(3125000h^3t^7 - 11375000h^3t^6 + 15487500h^3t^5 - 9362500h^3t^4 + 2100000h^3t^3 - 41490h^3t + 2218h^3)}{(504000h)}$$

$$\beta'_{\frac{3}{5}} = \frac{(3125000h^3t^7 - 10500000h^3t^6 + 12862500h^3t^5 - 6825000h^3t^4 + 1400000h^3t^3 - 26930h^3t + 1482h^3)}{(504000h)}$$

$$\beta'_{\frac{4}{5}} = - \frac{(1562500h^3t^7 - 4812500h^3t^6 + 5381250h^3t^5 - 2668750h^3t^4 + 525000h^3t^3 - 9825h^3t + 541h^3)}{(504000h)}$$

$$\beta'_1 = \frac{(312500h^3t^7 - 875000h^3t^6 + 918750h^3t^5 - 437500h^3t^4 + 84000h^3t^3 - 1545h^3t + 85h^3)}{(504000h)}$$

$$\alpha''_0 = \frac{25}{h^2}$$

$$\alpha''_{\frac{1}{5}} = - \frac{50}{h^2}$$

$$\alpha''_{\frac{2}{5}} = \frac{25}{h^2}$$

$$\beta''_0 = - \frac{(625ht^6)}{144} + \frac{(125ht^5)}{8} - \frac{(2125ht^4)}{96} + \frac{(125ht^3)}{8} - \frac{(137ht^2)}{24} + ht - \frac{(1243h)}{20160}$$

$$\beta''_{\frac{1}{5}} = \frac{(3125ht^6)}{144} - \frac{(875ht^5)}{12} + \frac{(8875ht^4)}{96} - \frac{(1925ht^3)}{36} + \frac{(25ht^2)}{2} - \frac{(2057h)}{11200}$$

$$\beta''_{\frac{2}{5}} = - \frac{(3125ht^6)}{72} + \frac{(1625ht^5)}{12} - \frac{(7375ht^4)}{48} + \frac{(2675ht^3)}{36} - \frac{(25ht^2)}{2} + \frac{(461h)}{5600}$$

$$\beta''_{\frac{3}{5}} = \frac{(3125ht^6)}{72} - 125ht^5 + \frac{(6125ht^4)}{48} - \frac{(325ht^3)}{6} + \frac{(25ht^2)}{3} - \frac{(2693h)}{50400}$$

$$\beta''_{\frac{4}{5}} = - \frac{(3125ht^6)}{144} + \frac{(1375ht^5)}{24} - \frac{(5125ht^4)}{96} + \frac{(1525ht^3)}{72} - \frac{(25ht^2)}{8} + \frac{(131h)}{6720}$$

$$\beta''_1 = \frac{(625ht^6)}{144} - \frac{(125ht^5)}{12} + \frac{(875ht^4)}{96} - \frac{(125ht^3)}{36} + \frac{(ht^2)}{2} - \frac{(103h)}{33600}$$

Shown below is the block and its derivatives which is been derived by evaluating (5) as well as its derivatives (6), (7) and (8) at the selected non-grid points.

$$\begin{aligned}
 & \begin{bmatrix} y_{n+\frac{1}{5}} \\ y_{n+\frac{2}{5}} \\ y_{n+\frac{3}{5}} \\ y_{n+\frac{4}{5}} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [y_n] + \begin{bmatrix} \frac{1}{5} \\ \frac{2}{5} \\ \frac{3}{5} \\ \frac{4}{5} \\ 1 \end{bmatrix} [hy'_n] + \begin{bmatrix} \frac{1}{50} \\ \frac{2}{25} \\ \frac{9}{50} \\ \frac{8}{25} \\ \frac{1}{2} \end{bmatrix} [h^2y''_n] \\
 & + h^3 \begin{bmatrix} \frac{13}{16676} & \frac{46}{46601} & -\frac{43}{56116} & \frac{13}{27052} & -\frac{16}{91325} & \frac{1}{36259} \\ \frac{97}{24097} & \frac{45}{4828} & -\frac{38}{7875} & \frac{59}{19042} & -\frac{89}{78750} & \frac{1}{5625} \\ \frac{111}{11341} & \frac{298}{10353} & -\frac{205}{26246} & \frac{49}{6487} & -\frac{55}{20013} & \frac{19}{43786} \\ \frac{255}{14102} & \frac{305}{5141} & -\frac{32}{5625} & \frac{158}{8837} & -\frac{8}{1575} & \frac{32}{39375} \\ \frac{233}{8064} & \frac{199}{1969} & \frac{5}{4032} & \frac{155}{8837} & -\frac{8}{1575} & \frac{32}{39375} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+\frac{1}{5}} \\ n + \end{bmatrix} \left| \begin{array}{c} 2 \\ 5 \end{array} \right| \left[fn + 35fn + 45fn + \right.
 \end{aligned}$$

(1)

$$\begin{bmatrix} y'_{n+\frac{1}{5}} \\ y'_{n+\frac{2}{5}} \\ y'_{n+\frac{3}{5}} \\ y'_{n+\frac{4}{5}} \\ y'_{n+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [y'_n] + \begin{bmatrix} \frac{1}{5} \\ \frac{2}{5} \\ \frac{3}{5} \\ \frac{4}{5} \\ 1 \end{bmatrix} [y''_n] + h^2 \begin{bmatrix} \frac{104}{10645} & \frac{207}{12089} & -\frac{761}{63000} & \frac{10}{1339} & -\frac{341}{126000} & \frac{57}{134243} \\ \frac{71}{3150} & \frac{18}{26057} & -\frac{37}{1575} & \frac{136}{7875} & -\frac{101}{15750} & \frac{8}{7875} \\ \frac{123}{3500} & \frac{438}{3503} & -\frac{9}{3500} & \frac{87}{2800} & -\frac{9}{875} & \frac{9}{5600} \\ \frac{179}{3749} & \frac{149}{824} & \frac{176}{7875} & \frac{587}{7603} & -\frac{16}{1575} & \frac{16}{7875} \\ \frac{61}{1008} & \frac{475}{2016} & \frac{25}{504} & \frac{125}{1008} & \frac{25}{1008} & \frac{11}{2016} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+\frac{1}{5}} \\ f_{n+\frac{2}{5}} \\ f_{n+\frac{3}{5}} \\ f_{n+\frac{4}{5}} \\ f_{n+1} \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} y''_{n+\frac{1}{5}} \\ y''_{n+\frac{2}{5}} \\ y''_{n+\frac{3}{5}} \\ y''_{n+\frac{4}{5}} \\ y''_{n+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [y''_n] + h \begin{bmatrix} \frac{19}{288} & \frac{461}{2326} & -\frac{133}{1200} & \frac{241}{3600} & -\frac{173}{7200} & \frac{3}{800} \\ \frac{14}{225} & \frac{43}{150} & \frac{7}{225} & \frac{7}{225} & -\frac{1}{75} & \frac{1}{450} \\ \frac{51}{800} & \frac{219}{800} & \frac{57}{400} & \frac{57}{400} & -\frac{21}{800} & \frac{3}{800} \\ \frac{14}{225} & \frac{64}{225} & \frac{8}{75} & \frac{64}{225} & \frac{14}{225} & \frac{1}{3654907200} \\ \frac{19}{288} & \frac{25}{96} & \frac{25}{144} & \frac{25}{144} & \frac{25}{96} & \frac{19}{288} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+\frac{1}{5}} \\ f_{n+\frac{2}{5}} \\ f_{n+\frac{3}{5}} \\ f_{n+\frac{4}{5}} \\ f_{n+1} \end{bmatrix} \quad (3)$$

Following Lambert (1973), the order of the method is computed to be $[6,6,6,6,6]^T$ with error constants $[-1/721080950, 1/1427572, -1/12911290, -1/1280789, -3/537908]^T$.

2.2 Convergence

The conditions required of a numerical method to be convergent is for it to be zero-stable and consistent as stated in Awoyemi (2003). Furthermore according to the theorem of Dahlquist (1979), the necessary and sufficient condition for a linear multi-step method to be convergent is to be consistent and zero-stable.

2.2.1 Zero-Stability

A linear multi-step method is said to be zero-stable if the roots r_w , $w=1,2,\dots, N$ of the first characteristics polynomial represented by $e(w) = \det(wA^* - B^*)$ satisfies $|r_w| < 1$ and the root $|w| = 1$ having multiplicity not exceeding one (Lambert 1973). Where

$$A^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B^* = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence $w=0,0,0,1$. This confirms the zero-stability of the new method.

2.2.2 Consistency

According to the theorem of Dahlquist (1979), a block method is said to be consistent if it has order $P \geq 1$. Hence, the order of this block is 6 as shown above.

Therefore, since the method (1) above satisfies the two conditions, hence it is convergent.

3. Test Problems

The following third order ODEs are considered.

Problem 1:

$$y''' = (-6y^4), y(1) = -1, y'(1) = -1, y''(1) = -2, h = 0.05$$

$$\text{Exact Solution: } y(x) = \frac{1}{x-2}$$

Source: Kayode and Obarhua (2017)

Problem 2:

$$y''' = y'(2xy'' + y'), y(0) = 1, y'(0) = \frac{1}{2}, y''(0) = 0, h = 0.1$$

$$\text{Exact Solution: } 1 + \frac{1}{2} \ln\left(\frac{2+x}{2-x}\right)$$

Source: Kayode and Obarhua (2017)

Problem 3:

$$y''' = -y, y(0) = 1, y'(0) = -1, y''(0) = 1, h = 0.1$$

Exact solution : $y(x) = \exp(-x)$

Source: Ogunware, Omole and Olanegan (2019)

Problem 4:

$$y''' = 3\sin x, y(0) = 1, y'(0) = 0, y''(0) = -2, h = 0.1$$

Exact Solution: $3\cos x + \frac{x^2}{2} - 2$

Source: Ogunware and Omole (2020)

The following acronyms are used in the Tables below:

ES - Exact Solution

CS - Computed Solution

NM - New Method

EINM - Error in New Method

EIKO (2017) - Error in Kayode and Obarhua (2017)

EIOOO (2019) - Error in Ogunware, Omole and Olanegan (2019)

EIOO (2020) - Error in Ogunware and Omole (2020)

Table 1: Comparison of EINM with EIKO (2017) for solving problem 1

x	ES	CS	$EINM$	$EIKO(2017)$
1.05	-1.052631578947368400	-1.052631578996598500	$4.923018E - 11$	$1.017989E - 09$
1.10	-1.111111111111111200	-1.111111108273341100	$2.837770E - 09$	$2.573804E - 08$
1.15	-1.176470588235294400	-1.176470573588322700	$1.464697E - 08$	$1.217663E - 07$

1.20	-1.250000000000000200	-1.249999955322418400	$4.467758E - 08$	$3.624816E - 07$
1.25	-1.333333333333333700	-1.333333225537460600	$1.077959E - 07$	$8.645410E - 07$
1.30	-1.428571428571429000	-1.428571200038967000	$2.285325E - 07$	$1.819265E - 06$
1.35	-1.538461538461539200	-1.538461089663978100	$4.487976E - 07$	$3.551325E - 06$
1.40	-1.666666666666667600	-1.666665823663091600	$8.430036E - 07$	$6.632889E - 06$
1.45	-1.818181818181819500	-1.818180269071991500	$1.549110E - 06$	$1.211591E - 05$
1.50	-2.000000000000001800	-1.999997164116821500	$2.835883E - 06$	$2.20E - 05$

Table 2: Comparison of EINM with EIKO (2017) for solving problem 2

x	ES	CS	$EINM$	$EIKO(2017)$
0.1	1.050041729278491400	1.050041726336692800	$2.941799E - 09$	$9.49E - 08$
0.2	1.100335347731075600	1.100335324572180800	$2.315889E - 08$	$1.32E - 06$
0.3	1.151140435936466800	1.151140354787355500	$8.114911E - 08$	$5.65E - 06$
0.4	1.202732554054082100	1.202732316954129100	$2.371000E - 07$	$1.58E - 05$
0.5	1.255412811882995200	1.255412189373310100	$6.225097E - 07$	$3.55E - 05$
0.6	1.309519604203111900	1.309518113193145000	$1.491010E - 06$	$6.97E - 05$
0.7	1.365443754271396200	1.365440449835604800	$3.304436E - 06$	$1.25E - 04$
0.8	1.423648930193601700	1.423642046266604000	$6.883927E - 06$	$2.11E - 04$
0.9	1.484700278594051700	1.484686594475364700	$1.368412E - 05$	$3.41E - 04$
1.0	1.549306144334054800	1.549279835286524500	$2.630905E - 05$	$5.32E - 04$

Table 3: Comparison of EINM with EIOOO (2019) for solving Problem 3

x	ES	CS	$EINM$	$EIOOO(2019)$
0.1	0.000000000061733174	0.904837417974226340	$6.173317E - 11$	4.038414E-11
0.2	-0.000000000429155489	0.818730753507137310	$4.291555E - 10$	2.576643E-10
0.3	-0.000000002239494590	0.740818222921212350	$2.239495E - 09$	5.283249E-10
0.4	-0.000000006061970814	0.670320052097610140	$6.061971E - 09$	2.272178E-10
0.5	-0.000000012520410153	0.606530672233043580	$1.252041E - 08$	2.416383E-10
0.6	-0.000000022173737735	0.548811658267764240	$2.217374E - 08$	1.468228E-09
0.7	-0.000000035518750330	0.496585339310159860	$3.551875E - 08$	4.085009E-09
0.8	-0.000000052991817823	0.449329017109039440	$5.299182E - 08$	6.779826E-09
0.9	-0.000000074969630059	0.406569734710229220	$7.496963E - 08$	1.099445E-08
1.0	-0.000000101769093375	0.367879542940535710	$1.017691E - 07$	2.052008E-09

Table 4: Comparison of EINM with EIOO (2020) for solving problem 4

x	ES	CS	$EINM$	$EIOO(2020)$
0.1	-0.000000000003778866	0.990012495837855890	$3.778866E - 12$	$3.9587E - 11$
0.2	-0.000000000026396774	0.960199733550121890	$2.639677E - 11$	$2.5377E - 10$
0.3	-0.000000000067629125	0.911009467444447220	$6.762912E - 11$	$6.3689E - 10$
0.4	-0.000000000127062805	0.843182982135718180	$1.270628E - 10$	$1.4670E - 09$
0.5	-0.000000000204103845	0.757747685875222120	$2.041038E - 10$	$2.5885E - 09$
0.6	-0.000000000297981861	0.656006845027017120	$2.979819E - 10$	$4.3061E - 09$
0.7	-0.000000000407758938	0.539526562261224420	$4.077589E - 10$	$6.4768E - 09$

0.8	-0.000000000532337618	0.410120128573833730	$5.323376E - 10$	$9.3677E - 08$
0.9	-0.000000000670471734	0.269829905482465160	$6.704717E - 10$	$1.2854E - 08$
1.0	-0.000000000820780929	0.120906918425200220	$8.207809E - 10$	$1.7155E - 08$

4. Discussion of Results

In Tables 1 and 2, the performance of the NM is revealed as it compares favourably with EIKO (2017) in spite of the higher step-number k considered for solving Problems 1 and 2. Similarly, the effectiveness of the NM is demonstrated in Table 3. The numerical results generated when the NM is applied to problem 3 are better when comparison was made with EIOOO (2019). In addition, Table 4 shows the excellent behaviour of the NM. The error proved the supremacy of the new developed method over EIOO (2020) for solving problem 4.

5. Conclusion

In this article, new numerical method capable of solving general third order initial value problems of ODEs is developed. Interpolation and collocation technique is adopted in deriving the new method which is of order six. The method is zero stable, consistent and therefore satisfies conditions for convergence. The effectiveness of the method is tested on some third order initial value problems and its accuracy is better when compared to some existing methods presented in literature.

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