



Unsteady MHD Poiseuille Oscillatory Flow between Two Infinite Parallel Porous Plates in an Inclined Magnetic Field with Suction Effect

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Abstract

The unsteady MHD poiseuille oscillatory flow between two infinite parallel porous plates in an inclined magnetic field with suction effect have been investigated. The governing equations, that is, the momentum, energy and species concentration equations have been written in dimensional form using the dimensionless parameters. Closed form method was employed to evaluate and solve the velocity profile U , temperature distribution θ , the species concentration C , skin frictions τ_1 and τ_2 , Nusselt numbers Nu_1 and Nu_2 and Sherwood numbers Sh_1 and Sh_2 . The results were graphically illustrated and tabulated. Among the results obtained, high Hartmann number suppresses the fluid velocity.

Keywords: Poiseuille, MHD, unsteady, magnetic field, porous, plates.

1. Introduction

Plasma is an example of electromagnetic fluid that can be created by heating a gas or subjecting it to a strong electromagnetic field, applied with a micro wave generator or laser. This decreases or increases the number of electrons, creating positive or negative charge particles called ions and is accompanied by the dissociation of molecular bond, the presence of a significant number of charges makes plasma electrically conductive so that it responds strongly to electromagnetic field. Like gas, plasma does not have a definite shape or a definite volume unless enclosed in a container, under the influence of a magnetic field, it may form structures such as filaments, beams and double layer. Heat transfer in the other hand can be said to be the transmission of energy from one region to another as a result of increase in temperature difference between them. While mass transfer can also be said to be the transportation or movement of materials or substances from a region of higher concentration to a region of lower concentration through a semi permeable membrane. Heat transfer occurs when there is temperature difference between two or more regions or surroundings. Much of the understanding of plasma came from the study of Magneto hydrodynamics (MHD) as

it is the study of interactions of electrically conducting fluids and electromagnetic fields. When fluid such as ionized gases (plasma, mercury and molten iron, electrolytes which are only but a few electrically conducting fluids, moves through a magnetic field, consequently a current is induced, and in turn the current interacts with the magnetic field to produce a body force on the fluid. Such forces (current) generated has been used in the generation of electricity by the help of (MHD) designed generators. The analysis of suction effect on fluids through magneto hydrodynamics has attracted many researches due to its application in geothermal and oil reservoir , where it deals with the behaviour of fluids under rest and motion, the phenomenon of unsteady Magneto-hydrodynamics (MHD) flow with suction effect have become very popular and a subject of growing interest due to its application in many engineering and geophysical processes such as cooling of nuclear reactor, geothermal reservoirs, underground energy transport, MHD pump ,MHD power generators. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself, which has some many applications in the field of engineering and science and as such as been a field of studies which have been attracting a considerable attention of engineers and scientists all over the world. It was Hannes Alfven (1942), a Swedish electrical engineer who first initiated the study of MHD. Shercliff (1956) considered the steady motion of an electrically conducting fluid in Pipes under transverse magnetic fields. Sparrow and Cess (1961) observed that free Convection heat transfer to liquid metals may significantly be affected by the presence of magnetic field. Drake (1965) considered flow in a channel due to periodic pressure Gradient and solved the resulting equation by separation of variables methods. Singh and Ram (1978) studied Laminar flow of an electrically conducting fluid through a channel in the presence of a transverse magnetic field under the influence of a periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. More to this, Ram et al (1984) analysed Hall effects on heat and mass transfer flow through porous media. Soundelgekar and Abdulla Ali (1986) studied the flow of viscous incompressible electrically conducting isothermal plate. Singh (1993) considered the steady MHD fluid flow between two parallel plates. John Mooney and Nick Stokes (1997) considered the effects of thermal radiation and free convection flow past a moving vertical plate. Al-Hadhrami (2003) discussed flow through horizontal channels of porous material and obtained velocity expressions in terms of the Reynolds number. Ganesh (2007) studied unsteady MHD Stokes flow of a viscous fluid between two parallel porous plates. Stamenkovic et al (2010) investigated MHD flow of two immiscible and electrically conducting fluids between isothermal, insulated moving plates in the presence of applied electric and magnetic fields. They matched the solution at the interface and it was found that decrease in magnetic field inclination angle flattens out the velocity and temperature profiles. Rajput and Sahu (2011) studied the effect of a uniform transverse magnetic field in the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel porous plates with constant temperature and variable mass diffusion. Mayonge et al (2012) studied steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field and discovered that high magnetic field strength decreases the velocity. Heat transfer effects

on rotating MHD coquette flow in a channel partially filled by a porous medium with hall current has been discussed by Singh and Rastogi (2012). Choudhary and DebH (2012) studied heat and mass transfer for viscoelastic MHD boundary layer flow past a vertical flat plate. Sandeep and Sugunamma (2013) analysed the effect of an inclined magnetic field in unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Joseph et al (2014) studied the unsteady MHD coquette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. They found out that when the magnetic field is high, it reduces the energy loss through the plate. Large Nusselt number corresponds to more active convection and high prandtl number decreases the temperature distribution. The unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer has been studied by Idowu et al (2014). Joseph et al (2015) studied the effect of heat and mass transfer on unsteady MHD Poiseuille flow between two infinite parallel Porous plates in an inclined magnetic field. In this present paper, we investigated the unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field with suction effect.

2. Formulation of the Problem

The concept of magneto hydrodynamics phenomenon can simply be described as follows: First, consider an electrically conducting fluid moving with velocity V . At right angles to this flow, we apply a magnetic field, the field strength of which is represented by the vector B . We assume that the fluid has attained unsteady state conditions. That is flow variables are dependent of the time t because of the interaction of the two fields, an electric field vector denoted E is induced at right angles to both V and B . This electric field is given by

$$E = V \times B \quad (1)$$

If we assume that the conducting fluid is isotropic/exhibits adiabatic flow in spite of the magnetic field, then we denote the electrical conductivity of the fluid by a scalar σ . By Ohm's law, the density of the current induced in the conducting fluid denoted J is given by,

$$J = \sigma E \quad (2)$$

$$J = \sigma (V \times E) \quad (3)$$

Simultaneously occurring with the induced current is the Lorentz force F given by

$$F = J \times B \quad (4)$$

This force occurs because, as an electric generator, the conducting fluid cuts the lines of the magnetic field. The vector F is the vector cross product of both J and B and is a vector perpendicular to the plane of both J and B . This induced force is parallel to V but in opposite direction. Laminar flow through a channel under uniform transverse magnetic field is important because of the use of MHD generator, MHD pump and electromagnetic flow meter. We now consider an electrically conducting, viscous, unsteady, incompressible fluid moving between two infinite parallel plates both kept at a constant distance $2h$ between them. Both plates of the channel are fixed with no motion. This is plane poiseuille flow. The equations of motion are the continuity equation.

$$\nabla \cdot V = 0 \quad (5)$$

And the Navier-Stokes equations

$$\rho \left[\left(\frac{\partial y}{\partial t} + V \cdot \nabla \right) V = f_B - \nabla p + \mu \nabla^2 V \right] \quad (6)$$

Where ρ is the fluid density, f_B is the body force per unit mass of the fluids, μ is the fluid viscosity and P is the pressure acting on the fluid. If we assume a one dimensional flow so that we choose the axis of the channel formed by the two plates as the x -axis and assume that the flow is in this direction. Observe that u , \bar{v} and w are the velocity components in \bar{x} , \bar{y} and \bar{z} directions respectively.

Then this implies

$$\bar{v} = w = 0$$

And $u \neq 0$, then the continuity equation is satisfied.

From this we infer that u is independent of \bar{x} . This makes the nonlinear term $[(V \cdot \nabla)V]$. In the Navier-Stokes equation vanish. We neglect body forces, f_B which are mainly due to gravity in the Navier-Stokes equations and replace them with the Lorentz force and from the assumption that the flow is one dimensional, it means that the governing equation for this flow is:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{F_x}{\rho} \quad (7)$$

Where $\nu = \frac{\mu}{\rho}$ is the kinematics viscosity and F_x is the component of the magnetic force in the direction of x -axis. Assuming unidirectional flow so that $\bar{v} = w = 0$ and $B_x = B_z = 0$, since magnetic field is along y -direction so that

$$V = iu \text{ and } B = B_0 j.$$

Where, B_0 is the magnetic field strength.

Now,

$$F_x = [(iu \times jB_0)] \times jB_0 \quad (8)$$

So that we have

$$\frac{F_x}{\rho} = -\frac{\sigma}{\rho} B_0^2 U \quad (9)$$

Then (7), becomes

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 U \quad (10)$$

From (10), when angle of inclination is introduced, we have

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 U \sin^2(\alpha) \quad (11)$$

Where α is the angle between V and B . Equation (11) is general, in the sense that, both fields can be assessed at any angle α for $0 \leq \alpha \leq \pi$.

Because of the porosity of the lower plate, the characteristic velocity V_0 is taken as a constant so as to maintain the same pattern of flow against suction and injection of the fluid in which it is moving perpendicular to the fluid flow. The origin is taken at the Centre of the channel and x , y coordinate axes are parallel and perpendicular to the channel walls respectively. The governing equations, that is; the momentum equation, the energy equation and the concentration equation are as follows:

The momentum equation is given as:

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = -v_0 \frac{\partial u}{\partial y} - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 U \sin^2(\alpha) + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (12)$$

Since the flow is isentropic, the energy equation is given as

$$\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q}{\partial y} \quad (13)$$

Where k is the thermal conductivity of the fluid, ρ is the density, C_p is the specific heat constant pressure and T is the temperature.

The concentration equation is given as

$$\frac{\partial c}{\partial t} + v_0 \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} - K'_c(C - C_\infty) \quad (14)$$

The q in (13) is called the heat flux. It is given by,

$$\frac{\partial q}{\partial y} = 4\alpha^2(T_\infty - T) \quad (15)$$

The boundary conditions are

$$U(y, t) = 0, T = T_\infty, C = C_\infty, \text{ at } t = 0, y = -L$$

$$U(y, t) = \frac{y}{L}, T = T_w, C = C_w, \text{ at } t > 0, y = L \quad (16)$$

In order to solve equations (12), (13) and (14) subject to the boundary conditions (16), we introduce the following dimensionless parameters:

$$U = \frac{uv}{L}, t = \frac{tL^2}{\nu}, y = yL, P = p\rho \frac{v^2}{L^2}, x = xL, \theta = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{\rho L^2 g\beta(T_w - T_\infty)}{\mu\nu}, C = \frac{C - C_\infty}{C_w - C_\infty}$$

$$G_c = \frac{\rho L^2 g\beta^*(C_w - C_\infty)}{\mu\nu}, Pr = \frac{\mu C_p}{k}, N^2 = \frac{4\alpha^2 L^2}{k}, Sc = \frac{\nu}{D}, S = \frac{v_0 L}{\nu}$$

Equations (12), (13) and (14) now becomes

$$\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial y} = \frac{\partial u}{\partial y} - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M^2 u + Gr\theta + G_c C \quad (17)$$

Where $M = m^* \sin\alpha$ and $m^* = LB_0 \sqrt{\frac{\sigma}{\mu}} = Ha$

$$\frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta \quad (18)$$

$$\frac{\partial c}{\partial t} - S \frac{\partial c}{\partial y} = \frac{1}{Sc} \frac{\partial^2 c}{\partial y^2} - k_c C \quad (19)$$

And the corresponding boundary conditions are

$$U(-1, t) = 0, \theta(-1, t) = 0, C(-1, t) = 0$$

$$U(-1, t) = 1, \theta(-1, t) = 1, C(-1, t) = 1 \quad (20)$$

3. Method of Solution

The momentum equation, energy equation and concentration equation can be reduced to the set of ordinary differential equation, which are solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the closed form as follows:

For purely oscillation,

$$\begin{aligned}
 U(y, t) &= U_0(y)e^{i\omega t} \\
 \theta(y, t) &= \theta_0(y)e^{i\omega t} \\
 C(y, t) &= C_0(y)e^{i\omega t} \\
 -\frac{\partial p}{\partial x} &= \lambda e^{i\omega t}
 \end{aligned}
 \tag{21}$$

By putting equation (21) into equations (17) to (20), we have the following ordinary differential equations

$$U_0''(y) + SU_0'(y) - (M^2 + i\omega)U_0(y) = -\lambda - Gr\theta_0(y) - GcC_0(y) \tag{22}$$

$$\theta_0''(y) + S\theta_0'(y) + (PrN^2 - i\omega Pr)\theta_0(y) = 0 \tag{23}$$

$$C_0''(y) + SS_cC_0'(y) - (K_cS_c + i\omega S_c)C_0(y) = 0 \tag{24}$$

The boundary conditions become

$$U(-1, t) = 0, \theta_0(-1, t) = 0, C_0(-1, t) = 0$$

$$U(-1, t) = 1, \theta_0(-1, t) = 1, C_0(-1, t) = 1 \tag{25}$$

Since, equation (22) is a coupled differential equation, we first solve equations (23) and (24).

Thus, the solution is as follows;

$$\theta_0(y) = A_1e^{m_1y} + A_2e^{m_2y} \tag{26}$$

$$C_0(y) = A_3e^{m_3y} + A_4e^{m_4y} \tag{27}$$

$$U_0(y) = A_5e^{m_5y} + A_6e^{m_6y} + K_1 + K_2e^{m_1y} + K_3e^{m_2y} + K_4e^{m_3y} + K_5e^{m_4y} \tag{28}$$

Therefore, the solution for velocity, temperature and concentration profile are as follows

$$U(y, t) = [A_5e^{m_5y} + A_6e^{m_6y} + K_1 + K_2e^{m_1y} + K_3e^{m_2y} + K_4e^{m_3y} + K_5e^{m_4y}]e^{i\omega t} \tag{29}$$

$$\theta(y, t) = (A_1e^{m_1y} + A_2e^{m_2y})e^{i\omega t} \tag{30}$$

$$C(y, t) = (A_3e^{m_3y} + A_4e^{m_4y})e^{i\omega t} \tag{31}$$

The Skin Friction

The shear stress at the lower boundary and is given by

$$\begin{aligned}
 \tau_1 = \frac{\partial u}{\partial y} \Big|_{y=-1} &= (A_5m_5e^{-m_5} + A_6m_6e^{-m_6} + K_2m_1e^{-m_1} + K_3m_2e^{-m_2} + K_4m_3e^{-m_3} + \\
 &K_5m_4e^{-m_4})e^{i\omega t}
 \end{aligned}
 \tag{32}$$

The skin friction at the upper boundary is

$$\begin{aligned}
 \tau_2 = \frac{\partial u}{\partial y} \Big|_{y=1} &= (A_5m_5e^{m_5} + A_6m_6e^{m_6} + K_2m_1e^{m_1} + K_3m_2e^{m_2} + K_4m_3e^{m_3} + \\
 &K_5m_4e^{m_4})e^{i\omega t}
 \end{aligned}
 \tag{33}$$

The Nusselt Number

This is the rate of heat transfer across the boundary.

At the lower wall is given by

$$N_{u1} = \frac{\partial \theta}{\partial y} \Big|_{y=-1} = (A_1 m_1 e^{-m_1} + A_2 m_2 e^{-m_2}) e^{i\omega t} \quad (34)$$

At the upper wall:

$$N_{u2} = \frac{\partial \theta}{\partial y} \Big|_{y=1} = (A_1 m_1 e^{m_1} + A_2 m_2 e^{m_2}) e^{i\omega t} \quad (35)$$

The Sherwood Number

This is the rate of mass transfer across the boundary at the lower boundary

$$S_{h1} = \frac{\partial c}{\partial y} \Big|_{y=-1} = (A_3 m_3 e^{-m_3} + A_4 m_4 e^{-m_4}) e^{i\omega t} \quad (36)$$

At the upper boundary

$$S_{h2} = \frac{\partial c}{\partial y} \Big|_{y=1} = (A_3 m_3 e^{m_3} + A_4 m_4 e^{m_4}) e^{i\omega t} \quad (37)$$

4. Data Presentation and Discussion of Results

4.1 Data Presentation

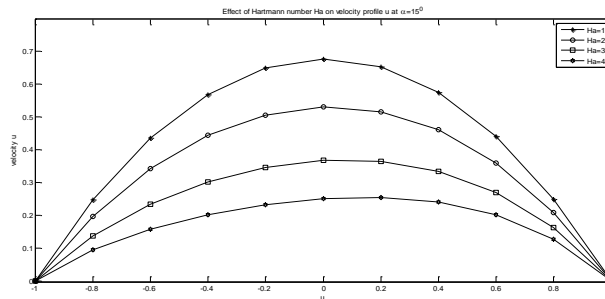


Figure 1: Effect of Hartmann number Ha on velocity u with $\alpha = 15^\circ$, $N = 1$, $t = 0.5$, $Gr = 1$, $Gc = 1$, $Sc = 1$, $Kc = 1$, $\omega = 1$ and $Pr = 0.71$

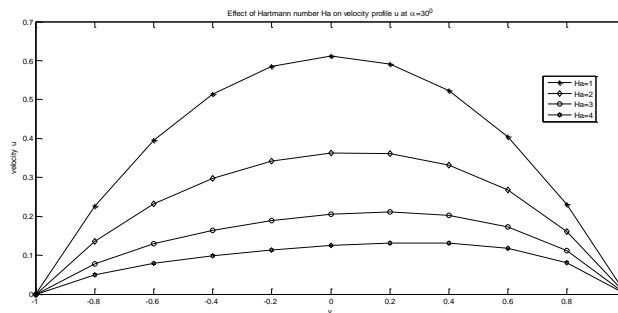


Figure 2: Effect of Hartmann number Ha on velocity u with $\alpha = 30^\circ$, $N = 1$, $t = 0.5$, $Gr = 1$, $Gc = 1$, $Sc = 1$, $Kc = 1$, $\omega = 1$ and $Pr = 0.71$

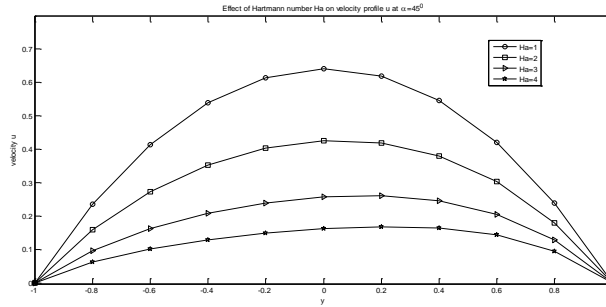


Figure 3: Effect of Hartmann number Ha on velocity u with $\alpha = 45^\circ, N = 1, t = 0.5, Gr = 1, Gc = 1, Sc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

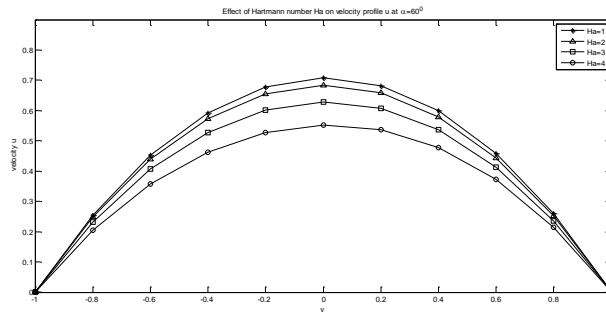


Figure 4: Effect of Hartmann number Ha on velocity u with $\alpha = 60^\circ, N = 1, t = 0.5, Gr = 1, Gc = 1, Sc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

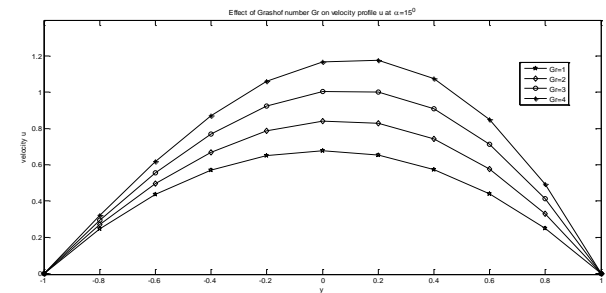


Figure 5: Effect of Grashof number Gr on velocity u with $\alpha = 15^\circ, N = 1, t = 0.5, Ha = 1, Gc = 1, Sc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

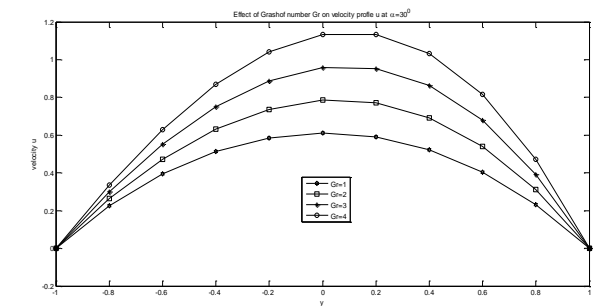


Figure 6: Effect of Grashof number Gr on velocity u with $\alpha = 30^\circ, N = 1, t = 0.5, Ha = 1, Gc = 1, Sc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

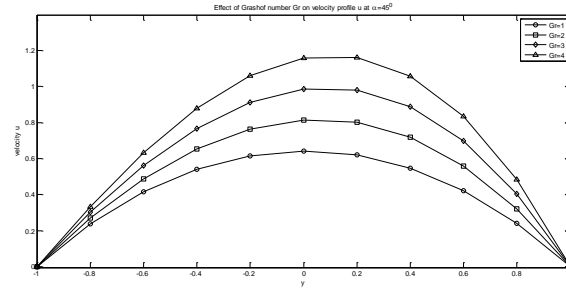


Figure 7: Effect of Grashof number Gr on velocity u with $\alpha = 45^\circ$, $N = 1$, $t = 0.5$, $Ha = 1$, $Gc = 1$, $Sc = 1$, $Kc = 1$, $\omega = 1$ and $Pr = 0.71$

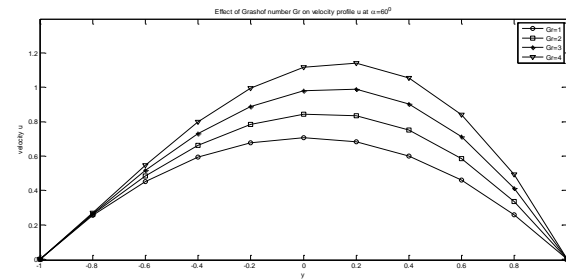


Figure 8: Effect of Grashof number Gr on velocity u with $\alpha = 60^\circ$, $N = 1$, $t = 0.5$, $Ha = 1$, $Gc = 1$, $Sc = 1$, $Kc = 1$, $\omega = 1$ and $Pr = 0.71$

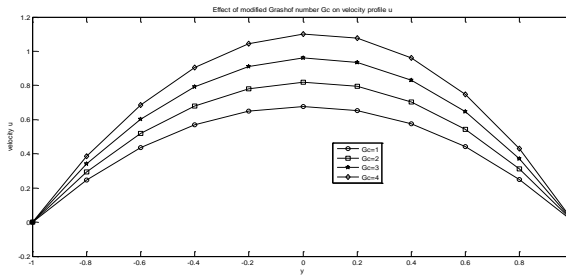


Figure 9: Effect of modified Grashof number Gc on velocity u with $\alpha = 15^\circ$, $N = 1$, $t = 0.5$, $Ha = 1$, $Gr = 1$, $Sc = 1$, $Kc = 1$, $\omega = 1$ and $Pr = 0.71$

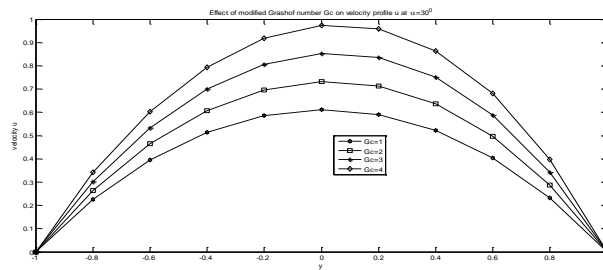


Figure 10: Effect of modified Grashof number Gc on velocity u with $\alpha = 30^\circ$, $N = 1$, $t = 0.5$, $Ha = 1$, $Gr = 1$, $Sc = 1$, $Kc = 1$, $\omega = 1$ and $Pr = 0.71$

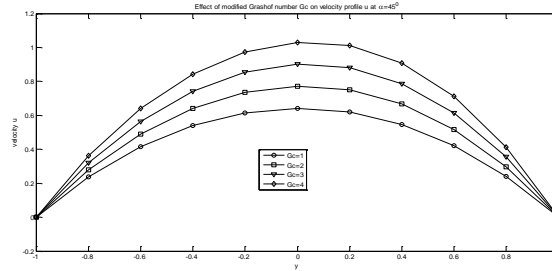


Figure 11: Effect of modified Grashof number Gc on velocity u with $\alpha = 45^{\circ}, N = 1, t = 0.5, Ha = 1, Gr = 1, Sc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

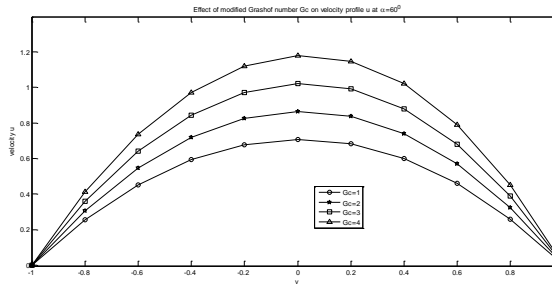


Figure 12: Effect of modified Grashof number Gc on velocity u with $\alpha = 60^{\circ}, N = 1, t = 0.5, Ha = 1, Gr = 1, Sc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

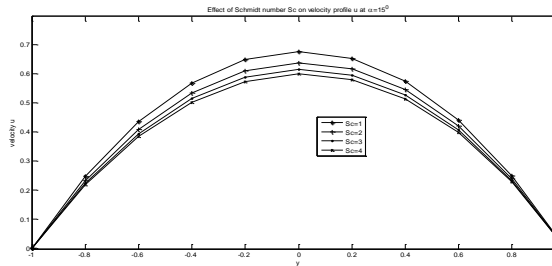


Figure 13: Effect of Schmidt number Sc on velocity u with $\alpha = 15^{\circ}, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

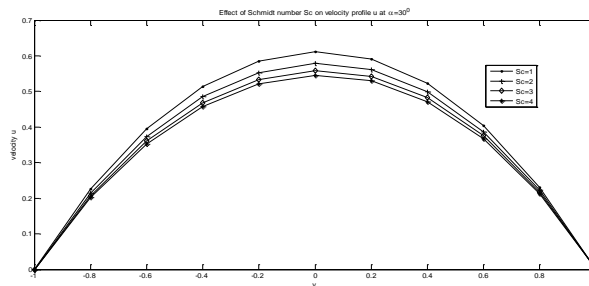


Figure 14: Effect of Schmidt number Sc on velocity u with $\alpha = 30^{\circ}, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

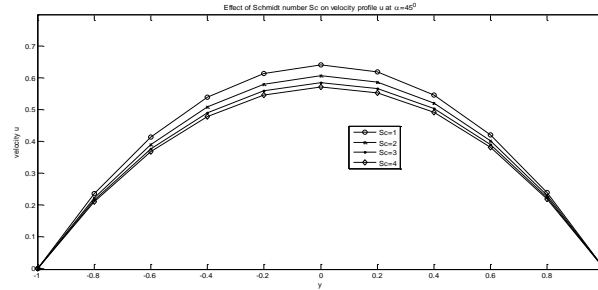


Figure 15: Effect of Schmidt number Sc on velocity u with $\alpha = 45^\circ, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

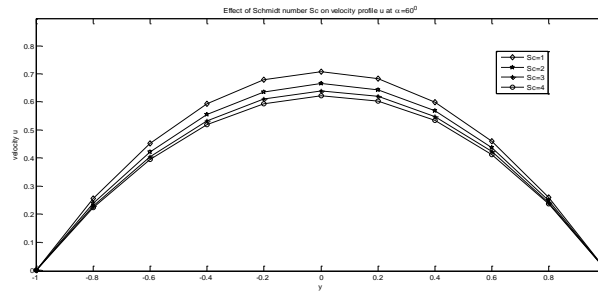


Figure 16: Effect of Schmidt number Sc on velocity u with $\alpha = 60^\circ, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Kc = 1, \omega = 1$ and $Pr = 0.71$

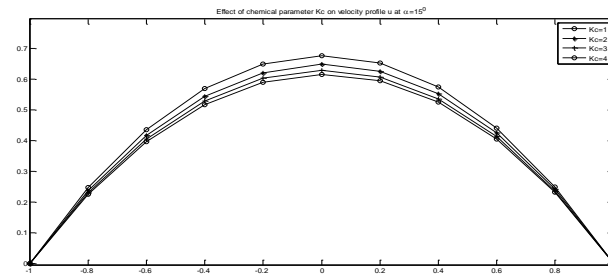


Figure 17: Effect of chemical parameter Kc on velocity u with $\alpha = 15^\circ, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Sc = 1, \omega = 1$ and $Pr = 0.71$

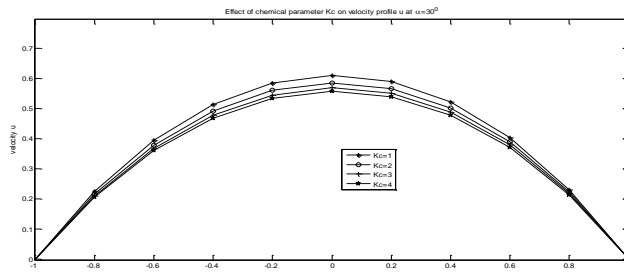


Figure 18: Effect of chemical parameter Kc on velocity u with $\alpha = 30^\circ, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Sc = 1, \omega = 1$ and $Pr = 0.71$

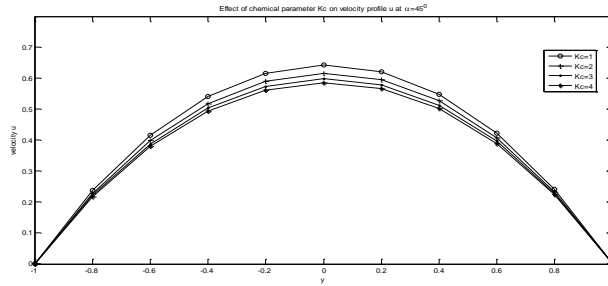


Figure 19: Effect of chemical parameter Kc on velocity u with $\alpha = 45^\circ, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Sc = 1, \omega = 1$ and $Pr = 0.71$

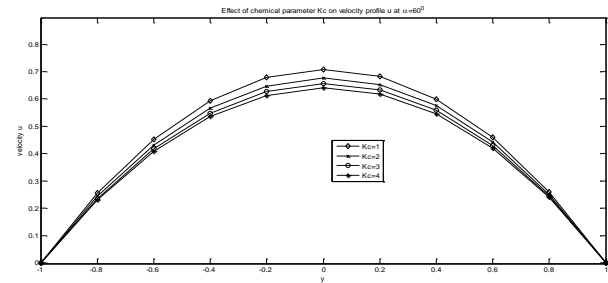


Figure 20: Effect of chemical parameter Kc on velocity u with $\alpha = 60^\circ, N = 1, t = 0.5, Ha = 1, Gr = 1, Gc = 1, Sc = 1, \omega = 1$ and $Pr = 0.71$

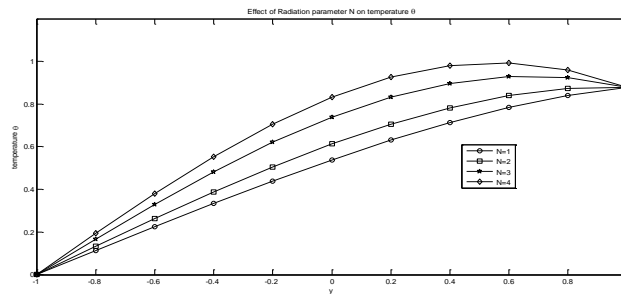


Figure 21: Effect of radiation parameter N on temperature θ with $\omega = 1, t = 0.5$ and $Pr = 0.71$

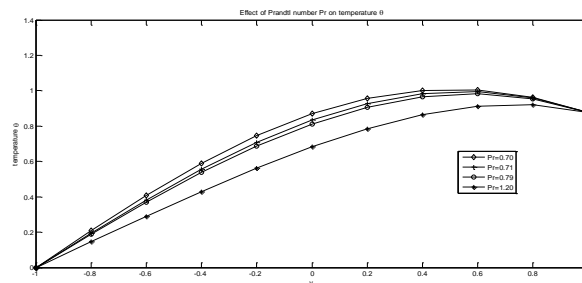


Figure 22: Effect of Prandtl number Pr on temperature θ with $\omega = 1, t = 0.5$ and $N = 1$

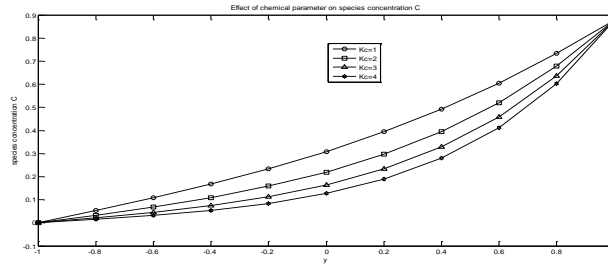


Figure 23: Effect of chemical parameter Kc on species concentration C with $t = 0.5, \omega = 1$ and $Sc = 1$

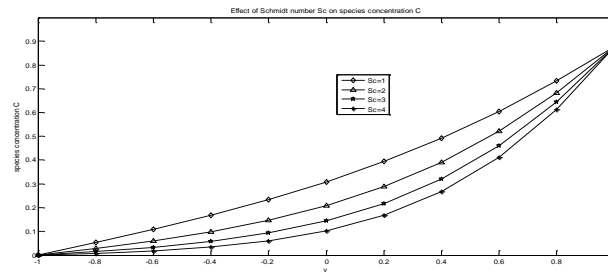


Figure 24: Effect of Schmidt number Sc on species concentration C with $t = 0.5, \omega = 1$ and $Kc = 1$

Table 1: variation of skin frictions τ_1 and τ_2 , Nusselt numbers Nu_1 and Nu_2 and Sherwood numbers Sh_1 and Sh_2 with time t .

t	τ_1	τ_2	Nu_1	Nu_2	Sh_1	Sh_2
0.1	0.1906	0.1898	1.8912	1.8224	0.0166	0.0166
0.2	0.2872	0.2563	2.0347	1.8241	0.0172	0.0172
0.3	0.3810	0.3202	2.1580	1.8075	0.0177	0.0177
0.4	0.4709	0.3810	2.2596	1.7729	0.0179	0.0179
0.5	0.552	0.4379	2.3387	1.7206	0.0180	0.0180
0.6	0.6358	0.4905	2.3944	1.6511	0.0180	0.0180
0.7	0.7092	0.5382	2.4262	1.5650	0.0177	0.0177
0.8	0.7754	0.5805	2.4338	1.4643	0.0173	0.0173
0.9	0.8339	0.6170	2.4170	1.3471	0.0167	0.0167
1.0	0.8840	0.6473	2.4761	1.2174	0.0159	0.0159

4.2 Discussion of Results

To discuss the effect of heat and mass transfer and suction effect on unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field with suction effects. The velocity profile u , the temperature distribution θ and the species concentration C are shown graphically against y using MATLAB for different values of the following parameters such as Hartmann number Ha , Grashof number Gr , modified Grashof number Gc , Radiation parameter N , Prandtl number Pr , Schmidt number Sc and chemical parameter Kc . Figures 1, 2, 3 and 4 present the effect of Hartmann number Ha on velocity U . It is inferred from these figures that an increase in the Hartmann number decreases the fluid velocity. Figures 5, 6, 7 and 8 shows the effect of thermal Grashof number Gr on velocity U . It is observed that an increase in thermal Grashof number Gr increases the velocity U . Figures 9, 10, 11 and 12 depict the effect of modified Grashof number Gc on velocity U . It is shown that the velocity increases as the modified Grashof number Gc increases. Figure 13, 14, 15 and 16 shows the effect of Schmidt number Sc on velocity U , it shows that an increase in the Schmidt number Sc entails an increase in the velocity U . Figure 17, 18, 19 and 20 illustrate the effect of chemical parameter Kc on velocity U , the velocity increases as the chemical parameters Kc increases. Figure 21 and 22 shows the effects of Radiation parameter N and Prandtl number Pr on temperature, it shows that an increase in both Radiation parameters and prandtl number increase the temperature distribution. Figure 23 and 24 describes the effect of chemical parameters Kc and Schmidt number Sc on the species concentration. It shows that an increase in both chemical parameter Kc and Schmidt number Sc decreases the species concentration. Table 1 depicts variation of skin frictions τ_1 and τ_2 , Nusselt numbers Nu_1 and Nu_2 and Sherwood numbers Sh_1 and Sh_2 with time t . It is observed that, as the time increases, the values varies/changes.

5. Conclusion

The unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field with suction effect have been investigated. The governing equations, that is, the momentum, energy and species concentration equations have been written in dimensional form using the dimensionless parameters. Closed form method was employed to evaluate and solve the velocity profile U , temperature distribution θ , the species concentration C , Skin frictions τ_1 and τ_2 , Nusselt numbers Nu_1 and Nu_2 and Sherwood numbers Sh_1 and Sh_2 .

The investigation of this research work leads to the following conclusions:

- At high Hartmann number, the velocity of fluid decreases.
- Velocity accelerates significantly due to increase in thermal Grashof number and modified Grashof number.
- Increase in Prandtl number increases the temperature distribution.
- Species concentration reduces with increase in chemical parameter and Schmidt number.

This research and its results would be useful in the field of engineer and sciences such as electrical engineering for design of (MHD) power generators and many more while it can be extended to the following:

- Unsteady MHD poiseuille flow between two infinite parallel porous plates in an incline magnetic field with viscous dissipation.
- Effect of Brinkman number on unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field.

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Nomenclature

B_0	Coefficient of electromagnetic field
C_f	Coefficient of skin friction
C_p	Specific heat capacity
Gr	Grashof number for heat transfer
g	Acceleration due to gravity
K	Permeability parameter of the porous medium
Nu	Nusselt number
P	Non-dimensional pressure gradient
Pr	Prandtl number
Re	Reynolds number
T'_{w1}	Fluid temperature at the upper wall
T'_{w2}	Fluid temperature at the lower wall
t	Time
Ha	Hartmann number
U	Velocity of the fluid
S	Suction parameter
u	Average velocity

Greek letters

ω	Frequency parameter
ωt	Periodic frequency parameter
β	Coefficient of volumetric thermal expansion for heat transfer
β^*	Coefficient of volumetric thermal expansion for mass transfer
ρ	Density of the fluid
μ	Dynamic viscosity
ν	Kinematic viscosity

Appendix

$$L_1 = N^2 - i\omega Pr$$

$$m1 = \frac{-S + \sqrt{S^2 - 4L_1}}{2}$$

$$m2 = \frac{-S - \sqrt{S^2 - 4L_1}}{2}$$

$$A_2 = \frac{-e^{-m1}}{e^{-(m1+m2)} - e^{(m2-m1)}}$$

$$A_1 = \frac{1 - A_2 e^{m2}}{e^{m1}}$$

$$L_2 = K_c S_c + i\omega S_c$$

$$m3 = \frac{-S + \sqrt{S^2 + 4L_2}}{2}$$

$$m4 = \frac{-S - \sqrt{S^2 + 4L_2}}{2}$$

$$A_4 = \frac{-e^{-m3}}{e^{-(m3+m4)} - e^{(m4-m3)}}$$

$$A_3 = \frac{1 - A_4 e^{m4}}{e^{m3}}$$

$$m5 = \frac{-S \pm \sqrt{S^2 - 4L_3}}{2}$$

$$m_6 = \frac{-S \pm \sqrt{S^2 - 4L_3}}{2}$$

$$K_1 = \frac{\lambda}{L_3}$$

$$K_2 = \frac{-G_r A_1}{m_1^2 + A m_1 - L_3}$$

$$K_3 = \frac{-G_r A_2}{m_2^2 + A m_2 - L_3}$$

$$K_4 = \frac{-G_c A_3}{m_3^2 + A m_3 - L_3}$$

$$K_5 = \frac{-G_r A_1}{m_4^2 + A m_4 - L_3}$$

$$L_4 = -(K_1 + K_2 e^{-m_1} + K_3 e^{-m_2} + K_4 e^{-m_3} + K_5 e^{-m_4})$$

$$L_5 = -(K_1 + K_2 e^{m_1} + K_3 e^{m_2} + K_4 e^{m_3} + K_5 e^{m_4})$$

$$A_6 = \frac{L_4 e^{m_5} - L_5 e^{-m_5}}{e^{m_5 - m_6} - e^{m_6 - m_5}}$$

$$A_5 = \frac{L_5 - A_6 e^{m_6}}{e^{m_5}}$$