



Fuzzy Logic Pricing Model for a New Product

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Abstract

Classical decision making process allows a decision maker to make decision from alternatives that comprises of actions, acts or strategies, state of nature, probability distribution, criterion for selecting a course of action, and sometimes Posteriori Probability Distribution is used in other to improve upon the choice made. However, the achievement or failure that a decision maker experiences is a function of the amount of information available and its degree of accuracy. In life, only few instances guarantee decisions to be made under complete information, such as the type of food to eat, where to go, who to be a friend to, who to marry, what kind of business venture to start, etc. However, these choices or decisions may be limited by certain conditions that are beyond our control in some instances. Most managerial decisions are made with some degree of uncertainty. Business tycoon, may authorize substantial financial investment with less than complete information about the demand of a product. It is possible to formulate or describe a lot of real life decision problems with elements of imprecision or uncertainty. In this paper, fuzzy sets and fuzzy logic are employed in making decision on the initial price of a rim of 500 sheets A4 75gsm 210mm x 297mm multipurpose duplicating paper based upon the cost of materials, labor, and production. The result obtained is quite promising as it lies between a tolerable range that yields profit and also satisfies customers. Fuzzy logic is therefore recommended for use especially when there is uncertainty, imprecision, vagueness, and ambiguity surrounding a decision making environment.

Keywords: Fuzzy Sets, Fuzzy Logic, Fuzzy Intersection

1. Introduction

The pricing of new products by producers, companies, businesses, wholesalers, retailers, etc. is a very difficult task. This task requires the joint effort and knowledge of several stakeholders in the industry. Inputs from experts in financial, management, marketing, and sales domain are necessary in recommending the opening value (price) of a new customer product. This is so because overpricing could give advantage to competitors and underpricing could lead to loss of profit which might lead to closure of business in the long run. When different economic agents are involved in the exchange of goods and services, pricing theory comes to play, as value is created and transferred between the agents. Price theory addressed a confusing issue concerning the “value” of a new product. Value is seen to possess dual meanings: first the usefulness of a product which is value in use; and secondly, the purchasing power which is the value in exchange. However, certain commodity’s value is a function of their relative scarcity and the cost of labor in acquiring them. This implies that labor represents the unit of exchange value. In Smith’s view the

minimal price of an item is a function of the currency used for the exchange and that is subject to changes (Weber, 2012).

In the eighteen century, Carl Menger, Leon Walras and William Stanley Jevons argued that the value of an item is the function of its marginal usefulness instead of labor. The intensity of labor needed to increase the quantity of an item controls the supply of the item. The earnest need for an item determine its supply and this earnest desire controls the price of the item. However, the concept of price (value) of an item to the excess demand and excess supply which is generally referred to Walrasian groping are integrated. The concept of price theory also makes provision for the determination of price by “rational preference”, where consumers have preference in their choices. Prices can also be determined by the behavior of market participants in a Walrasian equilibrium. In a stationary economy, there is the possibility of price adjustment due to uncertainty and Walrasian groping. The properties of Walrasian equilibrium are also destroyed due to externalities, and actions of agents. In general, the choice of a customer and the manufacturing decisions are usually a function of uncertainty. According to arbitrage pricing theory, the anticipated price of an item is better determined by linear grouping of vital macro-economic features like price indices (Ross, 1976; Roll & Ross, 1980). The prices of non-market goods such as public offices, organs of humans, etc. are decided through indifference by employing the welfare methods of Hicksian. Agents with market power can influence price in a general note, this is referred to as monopoly or oligopoly pricing. Economic reality agrees with the fact that information concerning an item for trading varies from one agent to the other, thereby yielding a condition of uncertainty in information. This can actually affect the price of an item.

Decision of individuals or organizations are usually prone to prejudice, that are either psychological or of imperfect reasoning capability which can lead to satisficing rather than optimizing (Simon, 1956; Camerer, 2006; Thaler, 1999).

Several decisions taken by managers are based on certain degree of uncertainty because the information used in making such decisions contain some elements of uncertainty, e.g. investment of funds on products can actually be on partial information, as it is practically impossible to know the exact demand for any product before manufacturing it. The growth of any business depends upon the right decisions made. Decision theory is a combination of two major business model techniques: descriptive and prescriptive business models in the classification of the extend of information. The extent of information may range from its absence to complete information, in between these extremes are uncertainty and risk factors. The success or failure of an individual, organization, or any cooperation largely depends on the quality of information, appropriate technique of analyzing the information and the right decision made.

Rommelfanger (2002) argued that decision making using fuzzy models are more realistic in real life decision situations and offer a reduction in the cost of information when a collaborative resolution process is utilised. This is basically due to the fact that conditions for real classical decision making techniques are usually not met in real life decision situations and the high cost associated with obtaining such information. Investigating recent theories in management, business and social sciences, a notable phenomenon is that they are built upon decision theory as stated in

Rommelfanger (2002). Conversely, experimental studies show that statistical decision approaches are rarely used in solving real life problems. The use of orthodox decision system in modeling a decision problem requires that the decision maker must identify the following features: a set of alternative acts, events corresponding to each act, a value that is either a gain or loss (result) that relates to each act-event grouping, a probability distribution for the chance of the event occurring, a criterion for selecting any course of action, and finally, to improve the chance of any conventional decision model, it is imperative to apply the concept of Posteriori probability distribution; that is by using extra information of a test environment. These are not only nonrealistic but equally not cost effective. Fuzzy logic (FL) however, is a more realistic way of modeling real life decision models because of its improvement on the flexibility of modeling human behavior and the human's real world (Ayuba, 2015; Peter & Tella, 2015). Human beings are not confident in most decisions they make, at times decisions are made base on anxiety or cynicism. When such limitation is not known, decision rules gotten from the observed data are usually misleading (Chen, Wang & Kuo, 2007). Human observations and way of reasoning typically includes some measures of uncertainty, or fuzziness (Engelbrecht, 2007). An xyz manufacturing business enterprise producing stationery papers is faced with the difficulty of deciding on the initial price to place on a rim of 500 sheets A4 75gsm 210mm x 297mm multipurpose duplicating paper. In the market, a rim of 500 sheets A4 75gsm 210mm x 297mm multipurpose duplicating paper ranges from six hundred and fifty naira (=N= 650.00k) only to seven hundred naira (=N= 700.00k) only depending on the manufacturing company. The xyz manufacturing company has three objectives designed by the marketing, sales, financial and management experts' after considering the cost of materials, labor, and production of a rim of 500 sheets A4 75gsm 210mm x 297mm multipurpose duplicating paper in the company as follows: 1. The rim price should be a moderate price, 2. The rim price should be a high price, and 3. The rim price should be close to competition price, and they added a constraint that the price must range between six hundred naira (=N= 600.00k) only to seven hundred and fifty naira (=N= 750.00k) only to ensure a marginal profit. Therefore, translating the real problem into a fuzzy model instead of selecting the best alternative is more realistic and finally yields a more reliable outcome (Oke & Charles, 2007; Fan & Feng, 2009). Section 2 introduces the theoretical frame work of marginal productivity theory of distribution, modern theory of distribution: demand and supply theory, and fuzzy set theory. Section3 introduces the problem of deciding on an initial price of a rim of paper. Section 4, gives the detailed methodology by introducing the concept of fuzzy logic for decision making environment. Section 5 discusses the results, while section 6 concludes the paper.

2. Theoretical Framework

2.1 Marginal Productivity Theory of Distribution

Marginal productivity theory of distribution is the neo-classical theory of distribution and is derived from Ricardo's "Marginal principle", Clark, Marshall and Hicks (as cited in Weber, 2012) are the main pro-pounders of the theory. The theory was initially propounded as an explanation

for the determination of wages that is the reward for labor but, it was later generalized as a theory of factor pricing for all the factors of production. “The theory states that the price of a factor of production is governed by its marginal productivity. The theory explained the process of equilibrium pertaining to the employment of input of various factors by an individual firm under perfect competition. In a perfectly competitive factor market, a firm can buy any number of units of factors of production, at the prevailing market price.” According to this theory, an entrepreneur or a firm will employ a factor at a given price till its marginal productivity tends to be equal to its price. It thus follows that the reward (price) of a factor tends to be equal to its marginal productivity. Marginal Productivity theory of distribution is the reward of a factor equals its marginal product. Marginal product, also known as marginal physical product, is the increment made to the total output by employing an additional unit of a factor, keeping all other factors constant. If the increase in the output is multiplied by the prevailing price of the product, the result is the marginal value product of that factor. The theory was criticized on the assumption that all units of factors are divisible and are been homogeneous. The criticism was overcome by Wicksteed as cited in Weber (2012) on the mathematical prove that all the marginal products of factor payments will add up to total product. Wicksteed revealed that aggregate production function must be linearly homogeneous before the condition would hold. The aggregate production function is given as:

$$Y = i(K, L, T) \quad (1)$$

Where Y is the total product; K is the capital; L is labor and T is technology. The equation above is homogeneous of degree one, then each factor was paid its marginal product. Then income share would indeed add up. Wicksteed proof was the sample of application of Euler’s Theorem. According to Euler’s Theorem if a function is homogeneous of degree r then

$$rY = iL^L + iK^K + iT^T \quad (2)$$

Where i is the first derivative of the function, r = 1 then

$$Y = iL^L + iK^K + iT^T \quad (3)$$

The marginal productivity of the distribution stated that if all factors are paid their marginal products, then the sum of factor income will add up to total product.

2.2 Modern Theory of Distribution Demand and Supply Theory

The modern theory of pricing which gives explanation of factor prices in the Demand and Supply Theory. The price of a commodity is determined by the demand for and supply of, a commodity, similarly the price of a productive service also is determined by demand for and supply of that particular factor. The demand for labor entirely depends upon the demand for goods. If the demand for goods increases, the demand for the factors which help to produce those goods will also increase. The demand for a factor of production will also depend on the quantity of the other factors required for the production. The demand price of a factor of production also depends upon the value of the finished product in the production of which the factor is used. The demand price of a commodity is normally higher, if more valuable is the finished product in which the factor is used.

The supply of labor entirely depends upon the size and composition of population, the occupational and geographical distribution, labor efficiency their training, expected income, relative preference for work and leisure, etc. Further, the supply of labor does not depend only on economic factors but many non-economic considerations also. Therefore, we can say that if the price of a factor increases, its supply will also increase and vice-versa. The graph below depicts the interaction between demand and supply of factors of production.

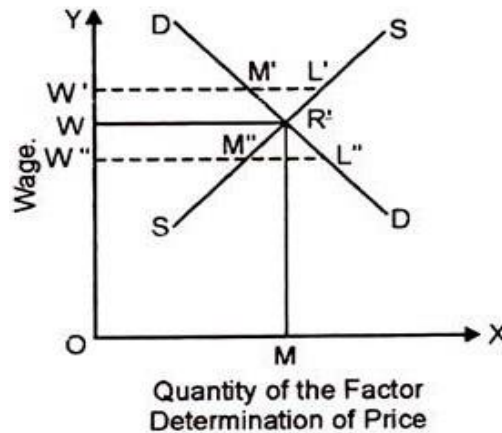


Figure 1: Price Prevalence in Equilibrium (www.Economicdiscussion.net, 2015)

The diagram above shows that price tends to prevail in the market at which the demand and supply are in equilibrium. This equilibrium is at the point of intersection of the demand and supply curves. The demand and supply curves intersect at the point R and the price of the factor will be OW at OW . Demand $W'M'$ is less than the supply WL' . In this case, competition among the sellers of the service tends to bring down the price to OW . On the other hand, at OW , "price the demand $W''L''$ is greater than the supply $W''M''$ ", hence price will tend to go up to OW at which the demand and supply will be equal (Jhingan, 2006; Saqib, 2021).

This study is hinged on the neo-classical theory of Marginal productivity theory of distribution, which states that if all factors are paid their marginal products, then the sum of factor income will add up to total product.

2.3 Fuzzy set Theory

In a traditional set, elements or members of a set are rigidly defined, that is, the elements are either in the set or are not contained in the set. A grade of membership of 1 is assigned to all elements that belong to a set and 0 is assigned to elements that are not in the set. There are no shades of gray areas in a classical set. Given a set from any universe of discourse, the classical set is only a subset of the universal set. The way humans really reason is not always exact as defined by the classical set. Observations and reasoning of humans usually includes some form of uncertainty. Approximate reasoning is what fuzzy set and fuzzy logic allows. In fuzzy sets, its members belong to the set at certain extent of certainty by extending the binary membership $[0, 1]$ of the classical

set to a range in the interval $[0, 1]$. In addition, unlike the classical set, all the members in the universe of discourse are elements of any given fuzzy set except that they belong to a certain degree. Fuzzy logic permits reasoning with such truths to infer new truths, with a grade of certainty linked with each fact. Fuzzy set and fuzzy logic permits the modeling or forming of common sense.

Gautama Buddha (563) and Buddhism are believed to be the pioneers of fuzzy logic as they habitually defined things in shades of gray. In the Western world, Aristotle is believed to give birth to fuzzy logic from the two-valued logic. Lukasiewicz work is the first to deviate from two-valued logic to three-valued logic and extended to random number of values later. Fuzzy sets are developed by Lotfi Zadeh who had an immense contribution to fuzzy logic (Konar, 2005).

3 Methodology

3.1 Concepts of Fuzzy Logic (FL)

FL is a logical system that extends the multi-valued logic, highlighting approximate way of reasoning in humans and deducing from the imprecise defined premises. FL is a principle of economics, precision is very expensive, only apply precision as far as it is necessary. The most relevant theories within FL are linguistic (fuzzy) variable and the fuzzy “If-Then” rule (Chen et al., 2007; Engelbrecht, 2007; Das & Sreedhar, 2012).

Linguistic Variables (LVs)

These are variables that carry values that are words, phrases, or sentences from a natural language. LVs are basically divided into three categories namely, quantification, occurrence, and prospect variables.

Quantification Variables (QVs): These are universal or existential variables that qualify nouns. E.g. all, some, few, most, many, none, etc.

Occurrence Variables (OVs): These are variables that signify the frequency of an event or activity. E.g. frequently, rarely, always, sometimes, etc.

Prospect Variables (PVs): These are variables that indicate or express the chance that an event can occur. E.g. likelihood, chance, possibility, etc.

LV is generally classified by a quintuple $\langle x, T(x), \cup, G, M \rangle$, Where x is the name of the variable, $T(x)$ is the set of x , that is, the set of names of linguistic values of x with each value being a fuzzy number (FN) defined on the universe of discourse $\cup$, G is a syntactic rule for generating the names of the values of x , and M is a semantic rule for associating with each value having a meaning.

Fuzzy Rules (FRs): Reasoning casually denotes generating inferences from a known set of realities and rules. These rules are a function of the knowledge and experience of experts in such domain. FRs are generally of the form “If antecedent(s) then consequent (s). The antecedent and consequent of FR is a proposition having LVs. Generally, FR is expressed as follows: *If P is p and Q is q then R is r.* Where *P and Q* are of the same universe of discourse, however, *R* is of a different universe of discourse.

Fuzzy Set (FS): FS is an ordered pair with the argument as the element from a given set and the entry as the membership function (MF) indicating the degree to which the argument belongs to the FS. Mathematically, FS is written as:

$$A = \{(x, \mu_A(x)) \mid x \in A, \mu_A(x) \in [0, 1]\} \quad (4)$$

Where $\mu_A(x)$ is the MF specifying the degree to which an element x belongs to the set A .

3.2 Intersection of Fuzzy Objectives and Constraints

Given any two FSs A and B , intersection of A and B is given as $A \cap B$ defined mathematically as:

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)), x \in \bigcup \quad (5),$$

where $\bigcup$ is the universe of discourse.

Generally, in decision making, objectives are expected to be met subject to fulfilling certain constraints. Let's consider decision making model comprising of a FS H with MF $\mu_H(x)$, constraint set C with MF $\mu_C(x)$, and x been an element of the discrete set of alternatives A .

According to Bellman & Zadeh (1970) the decision is a FS D having the MF $\mu_D(x)$, written as the intersection of H and C . Mathematically, it is expressed as:

$$D = H \cap C = \{(x, \mu_D(x)) \mid x \in [d_1, d_2], \mu_D(x) \in [0, 1]\} \quad (6)$$

The discrete set $[d_1, d_2]$ is obtained from a crisp set of alternatives A . The MF $\mu_D(x)$ specify the extent to which any $x \in [d_1, d_2]$ is a member of the decision D .

Using the MFs and equation (5), equation (6) yields equation (7)

$$\mu_D(x) = \min(\mu_H(x), \mu_C(x)), x \in A \quad (7)$$

Intersection operation is commutative, hence there is no need to distinguish between objectives and constraints, they can also be referred to as goals in general (Sivanandam, Sumathi, & Deepa, 2007; Bojadziev & Bojadziev, 2007).

Decision makers will always prefer discrete value as a result that optimizes the FS D . D has to be defuzzified, and that is best obtained from $[d_1, d_2]$ having the highest membership value in D set called maximizing decision expressed mathematically as:

$$x_{\max} = \{x \mid \max \mu_D(x) = \max \min(\mu_H(x), \mu_C(x))\} \quad (8)$$

Equations (3) -(5) are generalized for decision making models having several objectives and constraints. For m objectives and n constraints, the decision is:

$$D = H_1 \cap H_2 \cap \dots \cap H_m \cap C_1 \cap C_2 \cap \dots \cap C_n \quad (9)$$

The corresponding MF of D is

$$\mu_D(x) = \min(\mu_{H_1}(x), \dots, \mu_{H_m}(x), \mu_{C_1}(x), \dots, \mu_{C_n}(x)) \quad (10)$$

And the optimizing decision is $x_{\max} = \{x | \mu_D(x) \text{ is max}\} \quad (11),$

However, if the set of alternatives is not continuous, e.g. a subset of positive integers, the model is still valid.

4. Results and Discussions

Given the constraint as the set of alternatives defined as $A = \{x | 600 \leq x \leq 750\}$ the moderate (M) price as an objective defined on the set of alternatives A by

$$M = \mu_M(x) = \begin{cases} \frac{730-x}{150} & \text{for } 600 \leq x \leq 730 \\ 0 & \text{Otherwise} \end{cases} \quad (12)$$

H is the set of objective *high* price defined on the set of alternatives A as :

$$H = \mu_H(x) = \begin{cases} \frac{x-620}{30} & \text{for } 620 \leq x \leq 650 \\ \frac{680-x}{30} & \text{for } 650 \leq x \leq 680 \\ 0 & \text{Otherwise} \end{cases} \quad (13)$$

N is the set of objective *high* price defined on the set of alternatives A as :

$$N = \mu_N(x) = \begin{cases} \frac{x-650}{30} & \text{for } 650 \leq x \leq 680 \\ \frac{710-x}{30} & \text{for } 680 \leq x \leq 710 \\ 0 & \text{Otherwise} \end{cases} \quad (14)$$

Listing of elements: The number of elements and their respective membership to each set are listed for obtaining the decision interval, that is $[d_1, d_2] = [650, 680]$.

The set of moderate price

$$M = \{(600, 1.0), (650, 0.53)\} \quad (15)$$

The set of *high price*

$$H = \{(650, 1.0), (670, 0.33)\} \quad (16)$$

The set of close to competition price

$$N = \{(670, 0.67), (680, 1.0), (700, 0.33)\} \quad (17)$$

Given equations (10) and (11) and using the decision equation (4) gives

$$\mu_D(x) = \min(\mu_M(x), \mu_H(x), \mu_N(x)), x \in A \quad (18)$$

Solving the MFs within the decision interval, that is

$$\mu = \frac{680-x}{30} \text{ and } \mu = \frac{x-650}{30} \text{ yields the maximizing decision } X_{\max} = N665.00k.$$

Geometrical Representation

A schematic representation is shown in fig. 1. The decision region ranging from 650 to 680 naira on the horizontal axis representing the set of alternatives being the price in hundreds of naira is the intersection of the *high price* and *moderate price*. The maximum decision as seen on the horizontal axis is six hundred and sixty-five naira (=N= 665.00k) only.

The information and results obtained shows that decision making in a fuzzy environment using FL is much reliable and cost effective. The results from Figure 1 being the schematic representation of the objectives suggested that six hundred and sixty-five-naira (=N=665.00k) only be the initial unit selling price of a rim of 500 sheets A4 75gsm 210mm x 297mm multipurpose duplicating paper based upon the cost of materials, labor, and production.

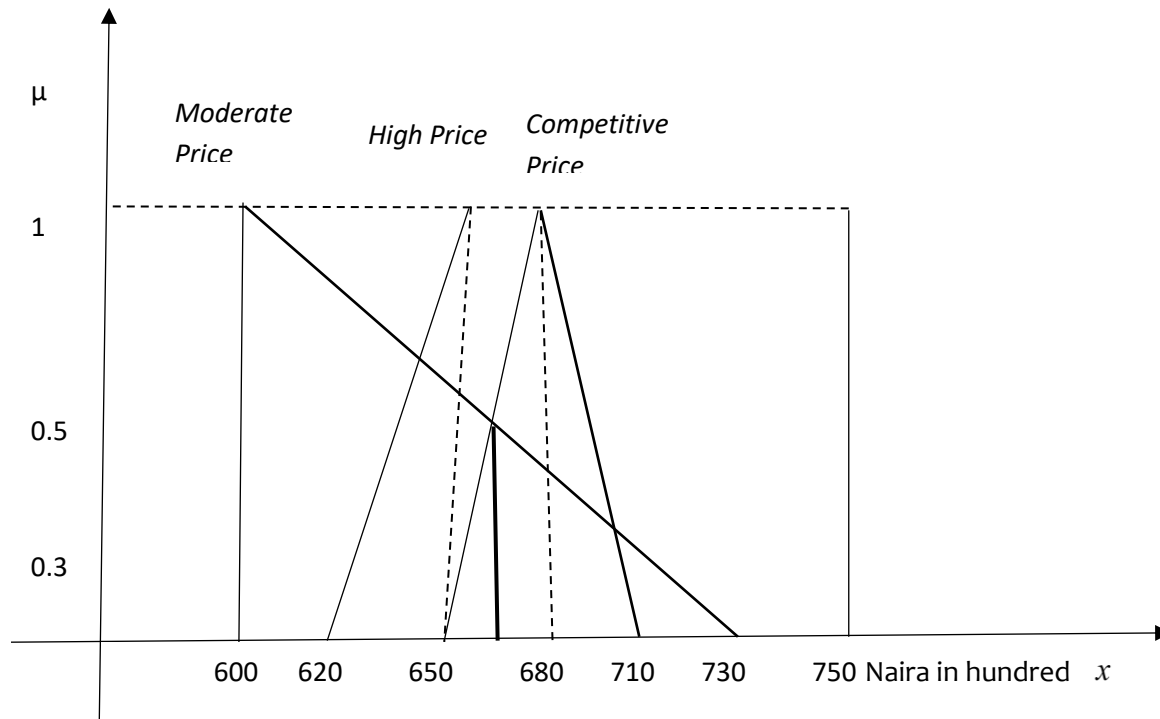


Figure 1. Pricing Model

5. Conclusion

In this study, fuzzy logic is applied to a xyz business company that produces stationeries and is saddled with the problem of determining the initial selling price of a 500 sheets A4 75gsm 210mm x 297mm multipurpose duplicating paper based upon the cost of materials, labor, and production. The model developed using the fuzzy logic intersection operator suggested a six hundred and sixty-five-naira (=N= 665.00k) only as the initial selling price per rim. This result agrees with the range of [650, 680] within which a marginal profit can be realized and also satisfies customers. In figure 1, the triangular number close to competition price contributes to the decision D , however, does not impact on the maximum decision. Only the moderate price and high price contributes to $x_{\max}$. A significant role is played by the close to competition price because its apex with the MF 1 is at $x = 680$ which determines the upper bound of the price range. The effect of moderate price yields a maximizing price of =N= 665.00k. Though the intersection operator is used in this illustration, there are other fuzzy logic approaches that can still be used like fuzzy averaging in making similar decision. In the future, fuzzy averaging will be applied in making decision in business, finance, and management.

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