



Seventh-Order Hybrid Block Algorithm for Direct Solution of Stiff Second Order Ordinary Differential Equations

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Abstract

Presented in this article is the derivation and application of an accurate hybrid block method for the direct computational solution of second order ordinary differential equations with initial value problems. The scheme was developed via collocation and interpolation of power series approximation to give a continuous linear multistep method. The evaluation of the continuous method at the grid and off grid points produced the discrete block method. The order, error constant, zero stability, consistency and convergence of the method were properly examined. The new block method generated more accurate result when compared with the work carried out by existing authors on the solution of stiff second order ordinary differential equations. The new method is therefore recommended for the solution of stiff second order ordinary differential equations. MAPPLE software was used for the computation.

Keywords: Power Series, Block Method, Implicit Method, Interpolation and Collocation; Stiff ODEs

1. Introduction

The computational solution of second order ordinary differential equations of the form

$$y'' = f(x, y, y'), y(a) = y_0, y'(a) = \beta \quad (1)$$

has been extensively carried out by several researchers using different methods like reduction approach, predictor-corrector method, Runge Kutta method and Taylor series approach. These researchers include but not limited to Olanegan *et al.* (2018), Waeleh (2017), Adoghe (2017), Abada (2017), Olanegan *et al.* (2015), Awoyemi *et al.* (2015). Wastage of human effort and computer time, complicated programming procedure, low order of the developed scheme, low level of accuracy and so on are the major setbacks associated with the aforementioned methods.

In order to overcome these challenges and bring improvement in numerical analysis, researchers such as Jator (2007), Awoyemi (2001), Omar and Kuboye (2016), Ogunware and Omole (2020), Akeremale *et al.* (2020), independently proposed the use of the block method for solving second order ODEs with IVPs directly whereby the accuracy of the block methods are far better than the methods used by the exiting authors. The uniqueness of block method is that in each application, the solution will be obtained concurrently at various points.

Hence, in this research the use of hybrid block method for the direct numerical solution of second order ODEs with IVPs will be our focus.

This article is partitioned into sections as follows: Abstract, Introduction, materials and method, Result and Discussion, Author's contribution, Acknowledgment and References.

2. Materials And Methods (Derivation of the Method)

Consider a power series of the form (2) as an approximate solution to equation (1)

$$y(x) = \sum_{j=0}^{(\phi+\theta)} a_j x^j \quad (2)$$

where ϕ and θ are the number of the interpolation and collocation points respectively.

The second derivative of (2) is obtained as

$$y'' = \sum_{j=2}^{(\phi+\theta)} j(j-1)a_j x^{j-2} \quad (3)$$

The combination of equations (1) and (3) yields the differential system

$$y'' = \sum_{j=0}^{(\phi+\theta)} j(j-1)a_j x^{j-2} = f(x, y, y') \quad (4)$$

Collocating (4) at $x = x_{n+j}$, $j = 0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}$ and interpolating (2) at $x = x_{n+j}$, $j = \frac{1}{8}, \frac{1}{4}$ gives arrangement of non-linear system in equation (5)

$$\begin{bmatrix} 1 & x_{n+1/8} & x_{n+1/8}^2 & x_{n+1/8}^3 & x_{n+1/8}^4 & x_{n+1/8}^5 & x_{n+1/8}^6 & x_{n+1/8}^7 \\ 1 & x_{n+1/4} & x_{n+1/4}^2 & x_{n+1/4}^3 & x_{n+1/4}^4 & x_{n+1/4}^5 & x_{n+1/4}^6 & x_{n+1/4}^7 \\ 0 & 0 & 2 & 6x_n & 24x_n^2 & 120x_n^3 & 720x_n^4 & 5040x_n^5 \\ 0 & 0 & 2 & 6x_{n+1/8} & 24x_{n+1/8}^2 & 120x_{n+1/8}^3 & 720x_{n+1/8}^4 & 5040x_{n+1/8}^5 \\ 0 & 0 & 2 & 6x_{n+1/4} & 24x_{n+1/4}^2 & 120x_{n+1/4}^3 & 720x_{n+1/4}^4 & 5040x_{n+1/4}^5 \\ 0 & 0 & 2 & 6x_{n+3/8} & 24x_{n+3/8}^2 & 120x_{n+3/8}^3 & 720x_{n+3/8}^4 & 5040x_{n+3/8}^5 \\ 0 & 0 & 2 & 6x_{n+1/2} & 24x_{n+1/2}^2 & 120x_{n+1/2}^3 & 720x_{n+1/2}^4 & 5040x_{n+1/2}^5 \\ 0 & 0 & 2 & 6x_{n+5/8} & 24x_{n+5/8}^2 & 120x_{n+5/8}^3 & 720x_{n+5/8}^4 & 5040x_{n+5/8}^5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} = \begin{bmatrix} y_{n+1/8} \\ y_{n+1/4} \\ f_n \\ f_{n+1/8} \\ f_{n+1/4} \\ f_{n+3/8} \\ f_{n+1/2} \\ f_{n+5/8} \end{bmatrix} \quad (5)$$

Finding the unknown $a_j, j = 0(1)7$ in equation (5) via Gaussian elimination procedures which are then filled into equation (2) to produce a continuous implicit scheme in equation (6)

$$y(t) = \alpha_{\frac{1}{8}}(t)y_{n+\frac{1}{8}} + \alpha_{\frac{1}{4}}(t)y_{n+\frac{1}{4}} + h^2 \left[\beta_0(t)f_n + \beta_{\frac{1}{8}}(t)f_{n+\frac{1}{8}} + \beta_{\frac{1}{4}}(t)f_{n+\frac{1}{4}} + \beta_{\frac{3}{8}}(t)f_{n+\frac{3}{8}} + \beta_{\frac{1}{2}}(t)f_{n+\frac{1}{2}} + \beta_{\frac{5}{8}}(t)f_{n+\frac{5}{8}} \right] \quad (6)$$

Now, using the transformation

$$t = \frac{x - x_n}{h}, \quad \frac{dt}{dx} = \frac{1}{h}$$

The coefficients of y_{n+j} and f_{n+j} are obtained in terms of t as:

$$\begin{aligned}
\alpha_{\frac{1}{4}} &= (-8t - 1) \\
\alpha_{\frac{1}{8}} &= (-8t + 2) \\
\beta_0 &= \left[-\frac{2048}{315}t^7 + \frac{256}{15}t^6 - \frac{272}{15}t^5 + 10t^4 - \frac{137}{45}t^3 + \frac{1}{2}t^2 - \frac{1609}{40320}t + \frac{3}{2560} \right] \\
\beta_{\frac{1}{8}} &= \left[+\frac{2048}{63}t^7 - \frac{3584}{45}t^6 + \frac{1136}{15}t^5 - \frac{308}{9}t^4 + \frac{20}{3}t^3 - \frac{13093}{80640}t + \frac{209}{15360} \right] \\
\beta_{\frac{1}{4}} &= \left[-\frac{4096}{63}t^7 + \frac{6656}{45}t^6 - \frac{1888}{15}t^5 + \frac{428}{9}t^4 - \frac{20}{3}t^3 + \frac{719}{20160}t + \frac{1}{3840} \right] \\
\beta_{\frac{3}{8}} &= \left[\frac{4096}{63}t^7 - \frac{2048}{15}t^6 + \frac{1568}{15}t^5 - \frac{104}{3}t^4 + \frac{40}{9}t^3 - \frac{247}{8064}t + \frac{7}{7680} \right] \\
\beta_{\frac{1}{2}} &= \left[-\frac{2048}{63}t^7 + \frac{2816}{45}t^6 - \frac{656}{15}t^5 + \frac{122}{9}t^4 - \frac{5}{3}t^3 + \frac{467}{40320}t - \frac{1}{2560} \right] \\
\beta_{\frac{5}{8}} &= \left[\frac{2048}{315}t^7 - \frac{512}{45}t^6 + \frac{112}{15}t^5 - \frac{20}{9}t^4 + \frac{4}{15}t^3 - \frac{149}{80640}t + \frac{1}{15360} \right]
\end{aligned} \tag{7}$$

Evaluating the continuous method at the non-interpolation points gives the following equations

$$y_{n+\frac{5}{8}} = 4y_{n+\frac{1}{4}} - 3y_{n+\frac{1}{8}} + \frac{h^2}{7680} \left[-f_n + 127f_{n+\frac{1}{2}} + 2f_{n+\frac{1}{4}} + 32f_{n+\frac{1}{8}} + 8f_{n+\frac{3}{8}} + 8f_{n+\frac{5}{8}} \right] \tag{8}$$

$$y_{n+\frac{3}{8}} = 2y_{n+\frac{1}{4}} - y_{n+\frac{1}{8}} + \frac{h^2}{15360} \left[-f_n - f_{n+\frac{1}{2}} + 194f_{n+\frac{1}{4}} + 24f_{n+\frac{1}{8}} + 24f_{n+\frac{3}{8}} \right] \tag{9}$$

$$y_{n+\frac{1}{2}} = 3y_{n+\frac{1}{4}} - 2y_{n+\frac{1}{8}} + \frac{h^2}{15360} \left[-2f_n + 22f_{n+\frac{1}{2}} + 412f_{n+\frac{1}{4}} + 47f_{n+\frac{1}{8}} + 242f_{n+\frac{3}{8}} - f_{n+\frac{5}{8}} \right] \tag{10}$$

$$y_n = 2y_{n+\frac{1}{8}} - y_{n+\frac{1}{4}} + \frac{h^2}{15360} \left[18f_n - 6f_{n+\frac{1}{2}} + 4f_{n+\frac{1}{4}} + 209f_{n+\frac{1}{8}} + 14f_{n+\frac{3}{8}} + f_{n+\frac{5}{8}} \right] \tag{11}$$

Evaluating the first derivative of the continuous scheme at all the points yields

$$y'_n = -\frac{1}{80640h} \left(645120y_{n+\frac{1}{8}} - 645120y_{n+\frac{1}{4}} + 3218h^2f_n - 934h^2f_{n+\frac{1}{2}} - 2876h^2f_{n+\frac{1}{4}} + 13093h^2f_{n+\frac{1}{8}} + 2470h^2f_{n+\frac{3}{8}} + 149h^2f_{n+\frac{5}{8}} \right) \tag{12}$$

$$y'_{n+\frac{1}{8}} = \frac{1}{80640h} \left(645120y_{n+\frac{1}{4}} - 645120y_{n+\frac{1}{8}} + 107h^2f_n - 277h^2f_{n+\frac{1}{2}} - 2710h^2f_{n+\frac{1}{4}} - 3104h^2f_{n+\frac{1}{8}} + 904h^2f_{n+\frac{3}{8}} + 40h^2f_{n+\frac{5}{8}} \right) \tag{13}$$

$$y'_{n+\frac{1}{4}} = -\frac{1}{80640h} \left(645120y_{n+\frac{1}{8}} - 645120y_{n+\frac{1}{4}} + 82h^2f_n - 262h^2f_{n+\frac{1}{2}} - 4444h^2f_{n+\frac{1}{4}} - 1355h^2f_{n+\frac{1}{8}} + 902h^2f_{n+\frac{3}{8}} + 37h^2f_{n+\frac{5}{8}} \right) \tag{14}$$

$$y'_{n+\frac{3}{8}} = -\frac{1}{80640h} \left(645120y_{n+\frac{1}{8}} - 645120y_{n+\frac{1}{4}} + 5h^2f_n + 389h^2f_{n+\frac{1}{2}} - 10058h^2f_{n+\frac{1}{4}} - 704h^2f_{n+\frac{1}{8}} - 4712h^2f_{n+\frac{3}{8}} - 40h^2f_{n+\frac{5}{8}} \right) \tag{15}$$

$$y'_{n+\frac{1}{2}} = -\frac{1}{80640h} \left(645120y_{n+\frac{1}{8}} - 645120y_{n+\frac{1}{4}} + 82h^2f_n - 4070h^2f_{n+\frac{1}{2}} - 8252h^2f_{n+\frac{1}{4}} - 1243h^2f_{n+\frac{1}{8}} - 11866h^2f_{n+\frac{3}{8}} + 149h^2f_{n+\frac{5}{8}} \right) \tag{16}$$

$$y'_{n+\frac{5}{8}} = -\frac{1}{80640h} \left(645120y_{n+\frac{1}{4}} - 645120y_{n+\frac{1}{8}} + 107h^2f_n + 14059h^2f_{n+\frac{1}{2}} + 11626h^2f_{n+\frac{1}{4}} + 32h^2f_{n+\frac{1}{8}} + 6280h^2f_{n+\frac{3}{8}} + 3176h^2f_{n+\frac{5}{8}} \right) \tag{17}$$

These schemes in equations (8) - (17) are merged together in matrix form and by using the matrix inversion, a block method of the following form is formed

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+1/8} \\ y_{n+1/4} \\ y_{n+3/8} \\ y_{n+1/2} \\ y_{n+5/8} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n-1/8} \\ y_{n-1/4} \\ y_{n-3/8} \\ y_{n-1/2} \\ y_n \end{bmatrix} + h \begin{bmatrix} 0 & 0 & 0 & 0 & 1/8 \\ 0 & 0 & 0 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 3/8 \\ 0 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 5/8 \end{bmatrix} \begin{bmatrix} y'_{n-1/8} \\ y'_{n-1/4} \\ y'_{n-3/8} \\ y'_{n-1/2} \\ y'_n \end{bmatrix} + \\
 & h^2 \begin{bmatrix} \frac{863}{129024} & -\frac{761}{161280} & \frac{941}{322560} & -\frac{341}{322560} & \frac{107}{645120} \\ \frac{17}{630} & -\frac{37}{4032} & \frac{17}{2520} & -\frac{101}{40320} & \frac{1}{2520} \\ \frac{3501}{71680} & -\frac{9}{8960} & \frac{87}{7168} & -\frac{9}{2240} & \frac{9}{14336} \\ \frac{89}{1260} & -\frac{11}{1260} & \frac{19}{630} & -\frac{1}{252} & \frac{1}{1260} \\ \frac{11875}{129024} & \frac{625}{32256} & \frac{3125}{64512} & \frac{625}{64512} & \frac{275}{129024} \end{bmatrix} \begin{bmatrix} f_{n+1/8} \\ f_{n+1/4} \\ f_{n+3/8} \\ f_{n+1/2} \\ f_{n+5/8} \end{bmatrix} + h^2 \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1231}{32250} \\ 0 & 0 & 0 & 0 & \frac{71}{8064} \\ 0 & 0 & 0 & 0 & \frac{123}{8960} \\ 0 & 0 & 0 & 0 & \frac{47}{2520} \\ 0 & 0 & 0 & 0 & \frac{1525}{64512} \end{bmatrix} \begin{bmatrix} f_{n-1/8} \\ f_{n-1/4} \\ f_{n-3/8} \\ f_{n-1/2} \\ f_n \end{bmatrix} \quad (18)
 \end{aligned}$$

Substituting the schemes that made up the block into equations (12) to (17), gives

$$y'_{n+\frac{1}{2}} = y'_n + h \left[\frac{7}{180} f_n + \frac{7}{180} f_{n+\frac{1}{2}} + \frac{1}{15} f_{n+\frac{1}{4}} + \frac{8}{45} f_{n+\frac{1}{8}} + \frac{8}{45} f_{n+\frac{3}{8}} \right] \quad (19)$$

$$y'_{n+\frac{1}{4}} = y'_n + h \left[\frac{7}{180} f_n - \frac{1}{120} f_{n+\frac{1}{2}} + \frac{7}{360} f_{n+\frac{1}{4}} + \frac{43}{240} f_{n+\frac{1}{8}} + \frac{7}{360} f_{n+\frac{3}{8}} + \frac{1}{720} f_{n+\frac{5}{8}} \right] \quad (20)$$

$$y'_{n+\frac{1}{8}} = y'_n + h \left[\frac{95}{2304} f_n - \frac{173}{11520} f_{n+\frac{1}{2}} - \frac{133}{1920} f_{n+\frac{1}{4}} + \frac{1427}{11520} f_{n+\frac{1}{8}} + \frac{241}{5760} f_{n+\frac{3}{8}} + \frac{3}{1280} f_{n+\frac{5}{8}} \right] \quad (21)$$

$$y'_{n+\frac{5}{8}} = y'_n + h \left[\frac{95}{2304} f_n + \frac{125}{768} f_{n+\frac{1}{2}} + \frac{125}{1152} f_{n+\frac{1}{4}} + \frac{125}{768} f_{n+\frac{1}{8}} + \frac{125}{1152} f_{n+\frac{3}{8}} + \frac{95}{2304} f_{n+\frac{5}{8}} \right] \quad (22)$$

$$y'_{n+\frac{3}{8}} = y'_n + h \left[\frac{51}{1280} f_n - \frac{21}{1280} f_{n+\frac{1}{2}} + \frac{57}{640} f_{n+\frac{1}{4}} + \frac{219}{1280} f_{n+\frac{1}{8}} + \frac{57}{640} f_{n+\frac{3}{8}} + \frac{3}{1280} f_{n+\frac{5}{8}} \right] \quad (23)$$

$$y'_{n+\frac{5}{8}} = y'_n + h \left[\frac{95}{2304} f_n + \frac{125}{768} f_{n+\frac{1}{2}} + \frac{125}{1152} f_{n+\frac{1}{4}} + \frac{125}{768} f_{n+\frac{1}{8}} + \frac{125}{1152} f_{n+\frac{3}{8}} + \frac{95}{2304} f_{n+\frac{5}{8}} \right] \quad (24)$$

3. Analysis of the Basic Properties of the Method

In verifying the accuracy and applicability of the proposed method, the basic properties of the method which include order, error constant, consistency, convergence and zero stability are as discussed below.

3.1 Order and Local Truncation Error

Adopting the method of Lambert (1973) method for finding the order of a numerical scheme also applied to equation (18). Hence, the new hybrid block method is of uniform order $[7 \ 7 \ 7 \ 7 \ 7]^T$

with Local Truncation Error of $\left[-\frac{33953}{362880} \quad \frac{41447}{762333} \quad \frac{2117}{32431} \quad -\frac{3133}{52677} \quad -\frac{1123}{17477} \right]^T$

3.2 Consistency

For a linear multistep method to be consistent, the following criteria must be met.

Condition 1: $\rho \geq 1$

Condition 2: $\sum_{j=0}^k \alpha_j = 0$ where $j = (0 \dots 2)$

Condition 3: $\rho'(r) = 0$ when $r = 1$

Condition 4: $\rho''(r) = 2!\sigma(r)$ when $r = 1$

where ρ and σ are the first and second characteristic polynomials of (9), applying these conditions to (9), the method was found to be consistent.

3.3 Zero Stability of the Block

Definition: The block is said to be zero stable if the roots $z_s, s=1,2,3,\dots,n$ of the characteristics polynomial $\rho(z)$ defined by $\rho(z) = \det(zA - E)$ satisfies $|z_s| \leq 1$ and the roots $|z_s| = 1$ is simple.

For the hybrid method,

$$A = \left[z \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right] = 0$$

$$A = z^5 - z^4 = 0, z = 0, 0, 0, 0, 1$$

Hence the block is zero stable. Lambert (1973) and Ogunware and Omole (2020)

3.4 Convergence

Theorem 1: Convergence Lambert (1973), Fatunla (1991)

The necessary and sufficient condition for a linear multistep method to be convergent is for it to be consistent and zero stable.

From the theorem above, our block method is convergent.

4. Numerical Experiments

The accuracy of the method for the direct solution of (1) is tested on some stiff second order ODE problems. The obtained results are shown on the tables.

Problem 1:

$$y'' = y', y(0) = 0, y'(0) = -1, h = 0.1$$

Exact Solution: $y(x) = 1 - e^x$

Problem 2:

$$y'' + 1001y' + 1000y = 0$$

$$y(0) = 1, y'(0) = -1, h = 0.05$$

Exact Solution: $y(x) = e^{-x}$

Table 1: Comparison of the result of the developed scheme with the result of Adoghe and Omole (2017) and Abada *et al.* (2017) for Problem 1

X	Exact Solution	Computed Solution	Error	Error in Adoghe and Omole (2017)	Error Abada (2017) <i>et al</i>
0.1	-0.001250781575622639	-0.001250781575622584	5.52e-17	1.11e-16	8.00e-13
0.2	-0.003757040047308369	-0.003757040047308429	6.07e-17	2.77e-16	1.40e-12
0.3	-0.006269572003761992	-0.006269572003762011	1.90e-17	5.55e-16	3.07e-12
0.4	-0.011313519223611346	-0.011313519223611285	6.07e-17	9.43e-16	4.55e-12
0.5	-0.008788393148316143	-0.008788393148316228	8.50e-17	9.99e-16	7.43e-12
0.6	-0.007528195444533870	-0.007528195444533939	6.93e-17	2.55e-15	1.01e-11
0.7	-0.010050167084167949	-0.010050167084168019	6.93e-17	6.88e-15	1.47e-11
0.8	-0.012578451540634417	-0.012578451540633879	5.37e-16	1.28e-14	1.91e-11
0.9	-0.015113064615718930	-0.015113064615078900	6.40e-13	1.90e-14	2.59e-11
1.0	-0.013844966011693938	-0.013844966011373763	3.20e-13	2.62e-14	3.28e-11

Table 2: Comparison of the result of the developed scheme with the result of Abada *et al* (2017) and Jator (2007) for Problem 2

x	Exact Solution	Computed Solution	Error	Error in Abada (2017) <i>et al</i>	Jator (2007)
0.1	0.998438720067590380	0.998438720067590380	0.00e+00	1.00e-14	6.98e-12
0.2	0.999219055096323920	0.999219055096323920	0.00e+00	3.00e-14	1.00e-12
0.3	0.999609451284012130	0.999609451284012130	0.00e+00	4.00e-14	7.88e-12
0.4	0.998828811377365460	0.998828811377365340	1.11e-16	5.00e-14	1.04e-11
0.5	0.998048781107475520	0.998048781107475410	1.11e-16	6.00e-14	6.32e-11
0.6	0.996490547573966490	0.996490547573814830	1.51e-13	7.00e-14	1.00e-11
0.7	0.997269359998249500	0.997269359998244620	4.88e-15	7.00e-14	9.36e-12
0.8	0.997658994437520710	0.997658994437520490	2.22e-16	7.00e-14	2.64e-12
0.9	0.996879877730208140	0.996879877730172170	3.59e-14	7.00e-14	1.06e-11
1.0	0.996101369470117510	0.996101369469654880	4.62e-13	7.00e-14	2.32e-11

6. Discussion of Results

The results produced by the proposed method for solving stiff second order ODEs problems 1 and 2 are displayed in table (1) and (2). Table 1 shows the results generated for problem 1 using the proposed method in comparison with the method of Adoghe and Omole (2017) and Abada *et al* (2017). The new method performs better than the methods of Adoghe and Omole (2017) and Abada *et al* (2017). It was observed that the proposed method converges faster in terms of errors. Presented in table 2 is the result produced by the developed scheme in comparison with the results of Abada *et al* (2017) and Jator (2007). In terms of errors, the result of the new method is more superior to the method of Abada *et al* (2017) and Jator (2007).

7. Conclusion

In this research, we have derived, analyzed and implemented a new hybrid block method to directly solve stiff second-order ODEs by adopting power series as the basis function. The generated results reveal that the developed hybrid block method is more accurate than the existing method. The new method is therefore recommended for the solution of stiff second order ODEs.

Authors Contributions

This research was jointly carried out by the three authors.

Acknowledgement

The authors wish to appreciate the editor and anonymous referee for their valuable comments and suggestions which have further improved the quality of this work.

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