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An Improvement on Vogel's Method to Feasible the Solution of Transportation Problem

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Abstract
Allocation of scarce resources has been one of man's oldest problems. Resources allocations has become a great challenge when there are various demands and supplies to be met up with. This research work proposes some new methods of solving balanced transportation problems. Algorithms for these new methods are also developed in this study. The method tested with numerical examples and the solutions are the same with that of Vogel's approximation method and minimum cost method. New method is proposed and give result that's exact compared with that of the minimum cost method and Vogel's approximation method. And the result obtained from the proposed method shows that the method can serve as an alternative method to minimum cost and Vogel's methods respectively.

Keyword: North west corner rule, least cost method, Vogel's approximation method, average cost method and average difference cost method

1. Introduction

Transportation theory is a name given to the study of optimal transportation and allocation of resources, formalized by the French mathematician Monge, (1871). In the 1930 Tolstoi first studied the transportation problem mathematically and published the work in the collection "*Transportation Planning Volume I* for the National Commissariat of Transportation of the Soviet Union", Kantorovich published his work "the Methods of Finding the Minimal Kilometer age in Cargo-transportation space." Tolstoi, again in 1939 illuminated his approach by applying to the transportation of salt, cement and other cargo. The research was extended along the railway network of the Soviet Union, by Singh, (2015). Major advances in transportation theory were made in the field during World War II by the Soviet/Russian Mathematician and Economist Kantorovich, (1942). Consequently, the problem as stated is sometimes known as the Monge–Kantorovich transportation problem. Kantorovich won the Nobel Prize for economics in 1975 for his work on the optimal allocation of scarce resources, the only winner of the prestigious award to come from the USSR.

Hitchcock, (1941) worked on the distribution of a production from several sources to numerous localities. Koopman, (1947) also worked on the optimum utilization of transportation system and used model of transportation, in activity analysis of production and allocation. To determine the extent of conformity between the computed schedule and the actual sailings analytical approach used by Koopmans, Tjalling C., Stanley Reiter (1951). Charnes and Cooper, (1961) mentioned about transportation in their book – Management Models and Industrial Applications of Linear Programming. Followed by Ijiri, (1965) who mentioned about transportation problem in his book-Management Goals and Accounting for Control. Klein (1967) developed a primal method for minimal cash flows with applications to the assignment and transportation problems. Hadley, (1972) also included transportation problem in his book: Linear Programming. Lee (1972) and Ignizio, (1976) used goal programming to solve transportation problem. Mackinnon James (1975) developed an algorithm for the generalized transportation problem. Moore, et al, (1978) performed analysis of a transshipment problem with multiple conflicting objectives. Kwak, (1979) developed a goal programming model for improved transportation problem solutions, followed by Kvanli, (1980) where the model was used on financial planning. Oheigeartaigh, (1982) developed a fuzzy transportation algorithm and Arthur, et al, (1982) worked on the multiple goal production and logistics planning in a chemical and pharmaceutical company. Olson, (1984) has compared four goal programming algorithm while, Goyal, (1984) worked on improving Vogel`s Approximation Method (VAM) for unbalanced transportation problems by subtracting or adding suitable constants to the rows and columns of the cost matrix. Kwak and Schniederjans, (1985) framed goal programming solutions to transportation problem with variable supply and demand requirement. Ahuja (1986) developed an Algorithm for minimax transportation problem. In the same manner, Romero (1986) has done a survey of generalized goal programming, also Currin, (1986) worked on the transportation problem with inadmissible routes. An Initial Basic Feasible Solution (IBFS) for the transportation problem are usually obtained by using the North-West corner rule, Miinimum Cost Method and Vogel's Approximation Method. In the work on best optimality condition has been checking, optimization of transportation problem on variables was remarkably been significant to various discipline Shraddah Mishra, (2017).

Romero, (1991) has written a book on "critical issues in goal programming", followed by Tamiz and Jones, (1995) who had done a review of goal programming and its applications. Hemaida and Kwak, (1994) developed a linear goal programming model for transshipment problem with flexible supply and demand constraints. Sharma et al, (1999) analyzed various applications of Multi-objective programming in MS/OR. Sun, (2002) worked on the transportation problem with exclusionary side constraints and branch and bound Algorithm. Schrijver, (2002) worked on the History of Transportation and Maximum flows while Okunbor, (2004) worked on the Management Decision Making for Transportation Problems through Goal Programming. The transportation problem in a housing material manufacturer and derive a satisfactory solution to the taking into account not only the degree of satisfaction with respect to objectives of the housing

material manufacturer but also those of two forwarding agents to which the housing material manufacturer entrusts transportation of products by Sakawa, Nishizaki, & Uemura, (2002)
Linear programming (LP) can be defined as a mathematical technique for determining the best allocation of a firm's limited resources to achieve optimum goal (Yahya, Garba, Ige, & Adeyosoye, 2012). Linear programming can also be implied in analyzing the profit maximization in a product mix bakery. Mathematical term are used in formulation of the problem and solved using computer software known as Linear Programming \Solver (LIPS). From the literature there is a need for comparative study of the transportation theory where transportation techniques will be compared in proffering solutions to the transportation problems. Allocation of scarce resources becomes a huge problem especially when the destination to make supply is more than one as such a choice have to be made. The effective resource allocation plays a very important aspect in life. The optimization in mathematics, computer science and economics and other fields is always referred to the choosing of the best element from some set of available alternatives Ahmad H.A., (2012). Transportation models is now gaining much attention because of its advantages using continuum modeling approach in its dealing with dense-network models, macroscopic problems, and initial phase planning. The continuum modeling approach is usually used to the determination of facility location, route choice, pedestrian flow and policy and socio-economic analysis Ho H. W & Wong, S.C. (2006). There are chiefly two types of transportation models- "the Balanced and Unbalanced." We shall restrict ourselves on only the balanced transportation problem in which the sum total demand and supply are the same. In the existing methods and the two proposed Algorithms are used to solve balanced transportation problems in this work. These proposed methods give feasible solutions to any balanced transportation problem. The methods in this work are effective in solving balanced transportation theory only.

2. Material and Method

Programming method

The transportation Problem is a special type of Linear Programming Problem where the objective is to transport various quantities of single identical goods that are initially stored at various origins, to different destinations. Vogel's Approximation Method is one of the most efficient methods to obtain basic feasible solution, which is near to the optimal solution. In order to check the optimality of the cost, we applied the optimality criteria: Therefore, there must be (**m+n-1**) that is number of allocated cells (m-number of rows, n-number of columns) and allocated to cells less than (**m+n-1**), which are degenerate transportation problem.
In this research work we shall propose two methods of the existing three methods to feasible solutions, with a technique of resolving degeneracy.
The three existing methods are:
i. North West Corner Method (Layman's Method)
ii. Least Cost Method (Business Man's Method)
iii. Vogel's Approximation Method (Operations Method)

Mathematical Modelling of Transportation Problem

Considering $\boldsymbol{m}$ sources Source1, Source2, ..., Source $\boldsymbol{m}$ with capacities $\boldsymbol{a_1}, \boldsymbol{a_2}, . . . , \boldsymbol{a_m}$ and $\boldsymbol{n}$-destinations (sinks) with requirement $b_1, b_2, . . . , b_m$ respectively. The transportation cost from i^{th} source to j^{th} destination is $C_{i,j}$ and the amount shipped is $X_{i,j}$. Therefore, the mathematical formulation of transportation problem is given as thus:

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{i,j} X_{i,j} \quad (1)$$

$$\sum_{j=1}^{n} X_{i,j} = 0, \;\; j = 1,2,....., m \quad (2)$$

Subjected to

$$\sum X_{i,j} = b_j \quad j = 1, 2,.....n \quad (3)$$

$X_{i,j} \geq 0$ for all $\boldsymbol{i}$ and $\boldsymbol{j}$

There are different methods used to obtain the feasible solution.as follows

Source/Destination	$\boldsymbol{D_1}$	$\boldsymbol{D_2}$	. . .	$\boldsymbol{D_n}$	Availability $\boldsymbol{a_i}$
$\boldsymbol{S_1}$	C_{11} x_{11}	C_{12} x_{12}	. . .	C_{1n} x_{1n}	a_1
$\boldsymbol{S_2}$	C_{21} x_{21}	C_{22} x_{22}	. . .	C_{2n} x_{2n}	a_2
⋮			...		⋮
$\boldsymbol{S_m}$	C_{m1} x_m	C_{m2} x_m		C_{mn} x_m	a_m
Requirement $\boldsymbol{b_j}$	b_1	b_2		b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Table 1: A General Scheme of Transformation Problem

$$\sum_{i=1}^{n} X_{ij} = b_j \quad j = 1,2,\cdots,n$$

$\sum_{i=1}^{m} X_{ij} = a_i = \sum_{j=1}^{n} X_{ij} = b_j, \quad j = 1 , 2, \dots , m$ (Also called ***rim*** conditions)

That is; the total capacity or supply must be equal to total requirement or demand.

For a feasible solution the transportation problem is said to be unique and exist $\sum_{ij=1}^{m} a_i = \sum_{j=1}^{n} b_j \;\; rim$

For any real number $\lambda_1 \neq 0$ such $x_{i,j} = \lambda_i b_j$ for all i and j, then the transportation problem is said to be at sufficient condition such that for

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = k$$

Then, $\sum_{j=1}^{n} x_{i,j} = \sum_{j=1}^{n} \lambda_i b_j = \lambda_i \sum_{j=1}^{n} b_j = \lambda_i k$ (4)

Hence, $\lambda_i = \frac{1}{k} \sum_{j=1}^{n} x_{i,j} = \frac{a_i}{k}$ for all i and j

Thus $x_{i,j} = \lambda_i b_j = \frac{a_i b_j}{k}$ (5)

Since $a_i > 0$ and $a_i > 0$ for all i and j, therefore, $a_i b_j / k \geq 0$and hence a feasible solution exists, i.e. $x_{i,j} \geq 0$.

The number of basic variables (positive allocations) in any basic feasible solution are m+n-1 (the number of independent constraint equations) is said to satisfy all the ***rim*** conditions (reference).
In the mathematical modelling of transportation problem, there are ***m*** rows (capacity or supply constraint equations) and ***n*** columns (requirement or demand constraint equations). Thus there are in total **m + n** constraint equations. But out of **m + n** constraint equations one of the equations is redundant and therefore eliminated. Thus, there are m+n-1 linearly independent equations which can be verified by adding all the **m** rows equations and subtracting from the sum the first n-1 column equations, thereby getting the last column equation.
Given as

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{i,j} - \sum_{j=1}^{n-1} \sum_{i=1}^{m} x_{i,j} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n-1} b_j \quad (6)$$

Where

$$\sum_{i=1}^{m} x_{in} = b_n; \qquad \sum_{i=1}^{m} x_{in} = \sum_{j=1}^{n} b_j \quad (7)$$

Average Cost Method (ACM)

One way for obtaining feasible allocation and solution is called the average cost method. This method as the name implies, implores the use of averages in choosing row or column to start the allocation with. The Algorithm below shows how the method is used.

The Algorithm

Step 1: From the transportation table, we determine the highest Cost sayC_h and lowest cost $C_l$say for each $i$row and $j$column. The ***average cost*** is calculated for each row or column by adding the lowest cost element in that row or column with the highest cost element in the same row or column. Then their sum is divided by 2. *i.e.* $\frac{C_n + C_l}{2}$

where C_h is the highest cots and C_l is the lowest costs
Write down the average costs below the rows and by the side of the columns of the table.

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Step 2: Select the row (column) with the lowest average cost and allocate as much as possible from the supply and requirement values to the cell having the minimum cost. *If there is a tie in the values of average cost, then check the next level average cost i.e., the next two minimum cost, the route having less averages will be chosen.*
This rule is extremely helpful in obtaining basic feasible solution in close proximity to the optimal solution.
Step 3: Adjust the supply and demand conditions for that cell. Eliminate those rows (columns) for which the supply and demand requirements are met.
Step 4: Repeat the steps from 1-3 with the reduced table unless all demand and supply becomes zero.
The initial basic feasible solution is obtain as.

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost
F_1	24	23	43	14	60	28.5
F_2	34	32	12	40	80	26
F_3	20	22	23	41	100	30.5
Demand	40	70	80	50	240	
Average cost	27	27	27.5	27.5		

Table 2: A General Average Cost Scheme

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost
F_1	24	23	43	14	60	28.5
F_2	34	32	12	40	0	26
F_3	20	22	23 (8)	41	100	30.5
Demand	40	70	0	50	240	
Average cost	27	27	27.5	27.5		

Table 3: A First Scheme of Average Cost Model

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost
F_1	24	23	43	14	60	28.5
F_2	34	32	12	40	0	26
F_3	20	22	23 (8)	41	60	30.5
Demand	0 (4)	70	0	50	240	
Average cost	27	27	27.5	27.5		

Table 4: A Second Scheme of Average Cost Model

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost
F_1	24	23	43	14	60	28.5
F_2	34	32	12	40	0	26
F_3	20	22	2[illegible] (8)	41	0	30.5
Demand	0 (4)	1[illegible] (6)	0	50	240	
Average cost	27	27	27.5	27.5		

Table 5: A Third Scheme of Average Cost Model

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost
F_1	24	23	43	14	10	28.5
F_2	34	32	12	4[illegible] (5)	0	26
F_3	20	22	2[illegible] (8)	41	0	30.5
Demand	0 (4)	1[illegible] (6)	0	0	240	
Average cost	27	27	27.5	27.5		

Table 6: A Fourth Scheme of Average Cost Model

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost
F_1	24	23	43	14	0	28.5
F_2	34	[illegible] (1)	12	4[illegible] (5)	0	26
F_3	20	22	23 (8)	41	0	30.5
Demand	0 (4)	0 (6)	0	0	240	
Average cost	27	27	27.5	27.5		

Table 7: A Fifth Scheme of Average Cost Model

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{i,j} X_{i,j}$$

$$Z = 23 \times 10 + 14 \times 50 + 12 \times 80 + 20 \times 40 + 22 \times 60 = 4010$$

Proposed Method Improvement

In this method for each source and destination, we compute a ‘difference’ rating which is the difference between the highest and the lowest cost in each of the routes for the source and destinations then divide by two. Allocation is then made to the row or column with the highest average difference.

Algorithm

Step 1: From the transportation table, we determine the average difference for each row and column. The average differences are calculated for each row (or column) by subtracting the lowest cost element in that row (column) from the highest cost element in the same row (column). Write down the average difference below the rows (aside the columns) of the table. $\frac{C_n + C_l}{2}$ where C_h is the highest costs and C_l is the lowest costs

Step 2: Select the row (column) with the highest average difference rating and allocate as much as possible from the supply and requirement values to the cell having the minimum cost. *If there is a tie in the values of average differences, then check the next level average differences i.e., the next two minimum cost, the route having more next average difference will be chosen. (New Rule for a tie of penalties)*

This rule is extremely helpful in obtaining basic feasible solution in close proximity to the optimal solution

Step 3: Adjust the supply and demand conditions for that cell. Eliminate those rows (columns) for which the supply and demand requirements are met.

Step 4: Repeat the steps 1-3 with the reduced table unless all demand and supply becomes zero

Thus, we obtain an initial basic feasible solution.

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost difference
F_1	24	23	43	14	60	14.5
F_2	34	32	12	40	80	14
F_3	20	22	23	41	100	10.5
Demand	40	70	80	50	240	
Average Difference cost	7	5	15.5	13.5		

Table 8: A General Scheme of Proposed Model Two

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average cost difference
F_1	24	23	43	14	60	14.5
F_2	34	32	12	40	0	14
F_3	20	22	23 (8)	41	100	10.5
Demand	40	70	0	50	240	
Average Difference cost	7	5	15.5	13.5		

Table 9: First Scheme Performance of the Proposed Model Two

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost difference
F_1	24	23	43	14	10	14.5
F_2	34	32	12	4 (5)	0	14
F_3	20	22	2 (8)	41	100	10.5
Demand	40	70	0	0	240	
Average Difference cost	7	5	15.5	13.5		

Table 10: Second Scheme Performance of the Proposed Model Two

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average Cost difference
F_1	24	23	43	14	10	14.5
F_2	34	32	12	4 (5)	0	14
F_3	20	22	(8)	41	0	10.5
Demand	0 (4)	1 (6)	0	0	240	
Average Difference cost	7	5	15.5	13.5		

Table 11: Third Scheme Performance of the Proposed Model Two

Factory/markets	D_1	D_2	D_3	D_4	Availability	Average cost difference
F_1	24	23	43	14	0	14.5
F_2	34	32 (1)	12	4 (5)	0	14
F_3	20	22	2 (8)	41	0	10.5
Demand	0 (4)	0 (6)	0	0	240	
Average Difference cost	7	5	15.5	13.5		

Table 12: Fourth Scheme Performance of the Proposed Model Two

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{i,j} X_{i,j}$$

$$Z = 23\times10 + 14\times50 + 12\times80 + 20\times40 + 22\times60 = 4010$$

3. Results and Discussion

METHOD	RESULT	NO. OF ITERATIONS
Northwest corner Rule	7030	6
Minimum cost	4010	5
Vogel approximation method	4010	5
New proposed method 1 (ACM)	4010	5
New proposed method 2 (ADM)	4010	5

Table 13: Comparing the Solutions Obtained by the three known Methods with the Results of the two New Proposed Methods.

North-West corner method gives quick solution because computations take short time but yields a bad solution because it is very far from optimal solution. Vogel's approximation method and Minimum-cost method is used to obtain the shortest road. Advantage of Vogel’s approximation method and Minimum-cost method is that they both yield the best basic solution giving initial solution very near to optimal solution but the solution of Vogel’s approximation method is slow because computations takes longer time. The cost of transportation with Vogel's approximation method and Minimum-cost method is less than North-West corner method.

4. Summary and Recommendation

From the results the ‘New Methods of Solving Balanced Transportation Problems with the Algorithms developed shows an improvement. The solutions obtain from numerical examples solved where compared with existing Vogel’s Approximation Method and Minimum Cost Method. The new proposed method give result that is exact with that of the minimum cost method and Vogel’s Approximation Method. The result obtained from the two proposed methods shows that one of these methods or may be both is an alternative method to minimum cost and Vogel’s methods respectively

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