



An Unsteady Slip Flow of a Fluid Pass a Porous Medium with Thermal Conductivity

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Abstract

An unsteady free convective slip flow of a fluid, incompressible and viscous pass a porous medium along a vertical plate that is infinite with an unsteady thermal conductivity was studied. The suction velocity was taken to be constant and the flow channel was assumed to be free from any magnetic effect. Some dimensionless parameters were used to reduce the governing equations to non-dimensional form leading to a set of partial differential equation. These equations were also converted to ordinary differential equation by perturbation method and then solved analytically. The effects of resulted various governing physical parameters were studied on the velocity of the fluid and the skin-friction. The numerical results were displayed on graphs and tables with the aid of MATLAB R2009b software. It was discovered that the slip parameter, thermal Grashof number and thermal conductivity speed up the fluid flow while the Prandtl number reduces it. The skin friction was also increased by the increasing buoyancy force and thermal conductivity but decreased by increasing slip parameter and Prandtl number.

Keywords: Porous medium, skin-friction, slip flow, suction velocity, thermal conductivity.

1. Introduction

The research on fluid dynamics is gaining more and attention as the day passes due to its wide range of applications in everyday life which cut across industries and several areas in engineering. The study involves liquids, gases and plasmas in motion. In meteorology (weather forecasting) for example, the fluid state at a given time is sampled and thermodynamics/fluid dynamics equations are used to estimate the state of fluid at any time in the future (Vishu, 2013). Also in Mechanical Engineering, fluid dynamics is invloved in turbomachinery which is the machine that transport energy between a fluid and rotor, including turbines and compressors (Vishu, 2013). Sharma and Bismeeta (2017) studied how variable thermal conductitivy and viscosity along side Dufour and Soret affect the flow of fluid along a vertical cone in a free convective manner. Masoud *et al* (2017) investigated a similarity solution for mixed-convection boundary layer nanofluid flow on an inclined permeable surface. Kareem and Salawu (2017) in their own study focused on a non-darcy porous medium with an inclined magnetic field. Taking dissipation into consideration, they

examined the variable viscosity, Dufour, variable thermal conductivity and Soret number on the medium of the flow. From horizontal plate, immersed in a permeable medium, Rabeti (2014) studied heat transfer of nanofluids subject to free convection. In a stretching field along the vertical axis, Jadav and Hazarika (2016) considered a magneto-hydrodynamics dusty fluid flow in presence of heat generation. He observed the variable thermal conductivity and viscosity effects on the flow. Rehena and Alim (2015) analyzed entropy generation by nanofluid with variable thermal conductivity and variable viscosity in a solar collector in form of a flat plate. Mahender and Srikanth (2015) initiated a magneto-hydrodynamics (MHD) mass flow, unsteady and free convective through a permeable vertical plate with viscous dissipation. Motsa *et al* (2014) investigated a flow of non-linear nanofluid with sinusoidal wall temperature over a heated surface along the vertical axis. Animasahun and Oyem (2014) worked on a flat surface, porous and non-darcy. They studied the effects of Soret, variable viscosity, Dufor and varying thermal conductivity (k) on the mass transfer flow over the surface. In view of the above mentioned studies, with a constant suction velocity, this work investigated an unsteady slip flow of a fluid pass a porous medium with thermal conductivity varying with time along an infinite vertical plate.

2. Mathematical Analysis

The heat transfer flow was reduced to unsteady one dimensional flow. This was done by making the flow infinite along vertical axis. The suction velocity is taking to be constant and the channel of flow is assumed to be free from any magnetic effect. The thermal conductivity was also considered to be varying with time while a slip boundary condition is imposed on the flow. With the above conditions and constant viscosity of the fluid, using Boussinesq's approximation, the equations in dimensional form governing the flow are given as:

Continuity Equation

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

Momentum Equation

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial y^*} = \nu \frac{\partial^2 v^*}{\partial y^{*2}} + g\beta(T^* - T_\infty) \quad (2)$$

Energy Equation

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\partial}{\partial y^*} \left\{ k \frac{\partial T^*}{\partial y^*} \right\} \frac{1}{\rho c_p} \quad (3)$$

Subject to:

$$u^* = L^* \left(\frac{\partial u^*}{\partial y^*} \right), \quad T^* = T_w^* + \varepsilon(T_w^* - T_\infty^*)e^{n^* t^*} \quad \text{at} \quad y^* = 0 \quad (4)$$

$$u^* \rightarrow 0, \quad T^* \rightarrow T_\infty^* \quad \text{as} \quad y^* \rightarrow \infty \quad (5)$$

where y^* , is the dimensional distance along the horizontal axis, t^* is the dimensional time. u^* is the component of dimensional velocity along y^* direction. v^* is the suction velocity. T^* is the

dimensional temperature, T_∞^* is the free stream dimensional temperature, T_w^* is the dimensional wall temperature. ϑ is the kinematic viscosity, g is the acceleration due to gravity, β and β^* are the volumetric coefficient of thermal expansion and concentration expansion respectively, C_p is the specific heat capacity, k is the thermal conductivity.

According to Mohammed (2013), the suction velocity is taken to be constant and given as

$$v^* = -V_0 \quad (6)$$

Moreover, in line with Akinpelu *et al* (2016) and Kareem (2017), the thermal conductivity can also be considered to be varying with time and it is given as:

$$k = k_\infty(1 + \alpha t) \quad (7)$$

Where k_∞ is the fluid free stream thermal conductivity, α is the variable thermal conductivity parameter and t is the time.

Following Sharma (2005) and Mohammed (2013), the following non-dimensional quantities are introduced.

$$n = \frac{n^* v}{V_0^2}, \quad y = \frac{V_0 y^*}{v}, \quad u = \frac{u^*}{V_0}, \quad t = \frac{t^* V_0^2}{v}, \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}$$

$$Gr = \frac{v g \beta (T_w^* - T_\infty^*)}{V_0^3}, \quad P_r = \frac{v \rho C_p}{k_\infty}, \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*} \quad (8)$$

Using equations (6) – (8), equations (2) – (5) were reduced to non-dimensional form as follows:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr \theta \quad (9)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \left\{ (1 + \alpha t) \frac{\partial^2 \theta}{\partial y^2} \right\} \quad (10)$$

Subject to the dimensionless boundary conditions:

$$u = h \left(\frac{\partial u}{\partial y} \right), \quad \theta = 1 + \varepsilon e^{nt} \quad \text{at } y = 0 \quad (11)$$

$$u \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (12)$$

where P_r is the Prandtl number, Gr is the thermal Grashof number, α is the thermal conductivity parameter.

3. Method of Solution

Equations (9) and (10) are second order partial differential equations. These equations are reduced to ordinary differential equations using regular perturbation method and then solved analytically. The velocity and temperature of the fluid in the neighbourhood of the plate are represented as follows:

$$u(y, t) = u_0(y) + \varepsilon e^{nt} u_1(y) \quad (13)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) \quad (14)$$

Substituting equations (13) and (14) into equations (9) and (10), and ignoring the higher order terms $o(\varepsilon)^2$, we obtain

$$\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} = -Gre^{m_1 y} \quad (15)$$

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - nu_1 = -Gre^{m_4 y} \quad (16)$$

$$(1 + \alpha t) \frac{\partial^2 \theta_0}{\partial y^2} + P_r \frac{\partial \theta_0}{\partial y} = 0 \quad (17)$$

$$(1 + \alpha t) \frac{\partial^2 \theta_1}{\partial y^2} + P_r \frac{\partial \theta_1}{\partial y} + nP_r \theta_1 = 0 \quad (18)$$

The boundary conditions can also be rewritten as:

$$u_0 = h \left(\frac{\partial u_0}{\partial y} \right), \quad u_1 = h \left(\frac{\partial u_1}{\partial y} \right), \quad \theta_0 = 1, \quad \theta_1 = 1 \quad \text{at } y = 0 \quad (19)$$

$$u_0 \rightarrow 0, \quad u_1 \rightarrow 0, \quad \theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (20)$$

Solving equations (15) – (18) subject to the boundary conditions (19) and (20), we obtained:

$$u_0 = A_5 e^{-y} + A_7 e^{m_1 y} \quad (21)$$

$$u_1 = A_9 e^{m_8 y} + A_{10} e^{m_4 y} \quad (22)$$

$$\theta_0 = e^{m_1 y} \quad (23)$$

$$\theta_1 = e^{m_4 y} \quad (24)$$

Therefore, the velocity and temperature distributions respectively are found to be:

$$u = A_5 e^{-y} + A_7 e^{m_1 y} + \varepsilon e^{nt} (A_9 e^{m_8 y} + A_{10} e^{m_4 y}) \quad (25)$$

$$\theta = e^{m_1 y} + \varepsilon e^{nt} (e^{m_4 y}) \quad (26)$$

Where,

$$m_1 = \frac{-P_r}{1 + \alpha t}$$

$$m_4 = - \left(\frac{P_r}{2(1 + \alpha t)} + \sqrt{\frac{P_r^2 + 4nP_r(1 + \alpha t)}{4(1 + \alpha t)^2}} \right)$$

$$m_8 = - \left(\frac{1}{2} + \sqrt{\frac{1 + 4n}{4}} \right)$$

$$A_5 = \frac{(1 - hm_1)A_7}{hm_5 - 1}$$

$$A_7 = \frac{-Gr}{m_1^2 + m_1}$$

$$A_9 = \frac{(1 - hm_4)A_{10}}{hm_8 - 1}$$

$$A_{10} = \frac{-Gr}{m_4^2 + m_4 - n}$$

Knowing the velocity field, the skin-friction can be determined which is given as:

$$C_f = \left(\frac{\partial u_0}{\partial y} + \varepsilon e^{nt} \frac{\partial u_1}{\partial y} \right)_{y=0} = \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

$$C_f = -A_5 + m_1 A_7 + \varepsilon e^{nt} (m_8 A_9 + m_4 A_{10}) \quad (27)$$

4. Results and Discussion

To properly illustrate the behaviours of the emerging physical parameters from the transient velocity, temperature, skin friction and Nusset number, namely: the Prandlt number (Pr), thermal Grashof number (Gr), variable thermal and conductivity parameter (α), the numerical values of these parameters were computed and then displayed on graphs. In line with some existing works like Nwaigwe (2010) and Akinpelu *et al* (2020) among others, the following default values were adopted for the physical parameters involved in the flow, unless otherwise stated.

" $Pr = 0.71$ ", " $\alpha = 0.10$ ", " $\varepsilon = 0.01$ ", " $t = 1.00$ ", " $n = 10.00$ ", " $Gr = 2.0$ ", " $h = 0.4$ "

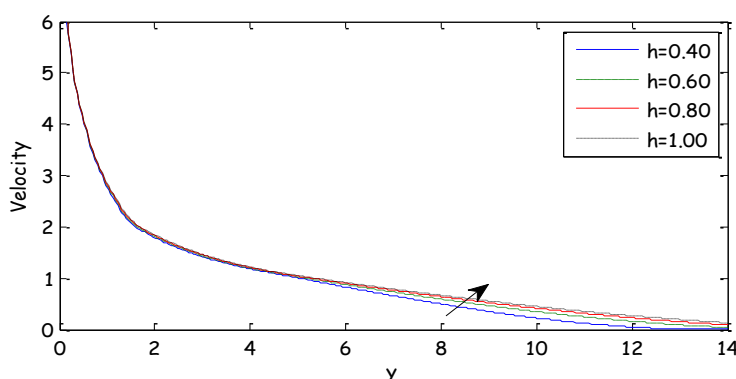


Figure 1: Velocity profile for increasing values of slip parameter, h with $Pr = 0.71, \alpha = 0.10, \varepsilon = 0.01, t = 1.00; n = 10.00, Gr = 2.0$

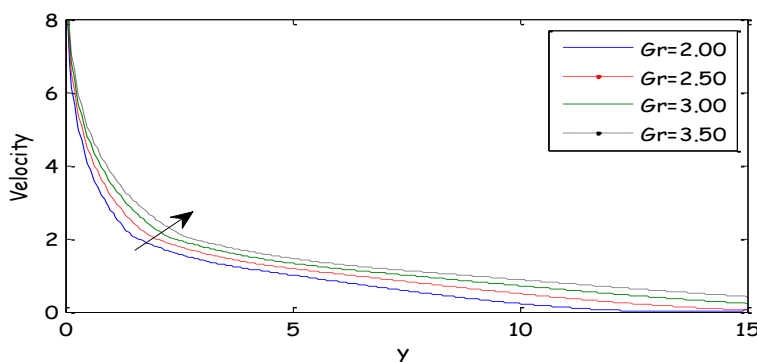


Figure 2: Velocity profile for increasing values of thermal Grashof number, Gr with $h = 0.40, \alpha = 0.10, \varepsilon = 0.01, t = 1.00, n = 10.00, Pr = 0.71$

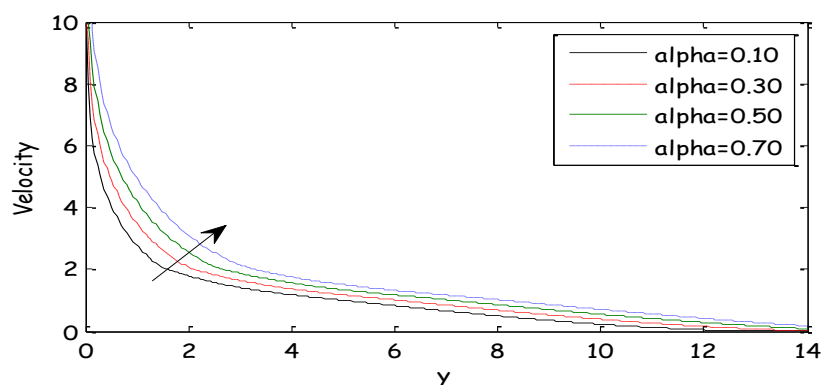


Figure 3: Velocity profile for increasing values of fluid's thermal conductivity, α with $Pr = 0.71, h = 0.40, \varepsilon = 0.01, t = 1.00, n = 10.00, Gr = 0.71$

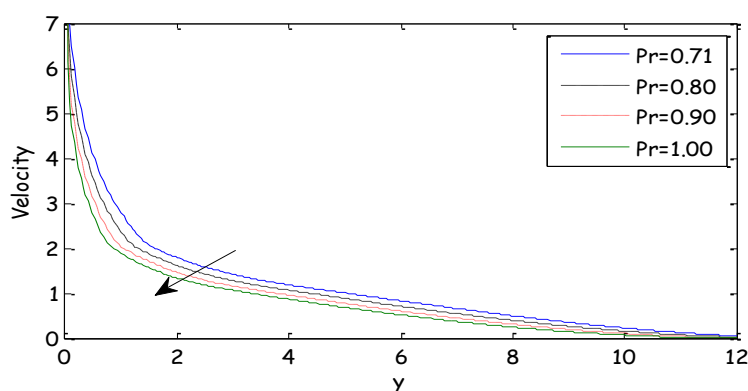


Figure 4: Velocity profile for increasing values of Prandtl number, Pr with $h = 0.40, \alpha = 0.10, \varepsilon = 0.01, t = 1.00, n = 10.00, Gr = 2.0$

Table 1: Effect of Prandtl number (P_r) on the skin-friction

P_r	A	ε	t	n	h	Gr	C_f
0.71	0.10	0.01	1.00	10.00	0.40	2.00	34.0090
0.75	0.10	0.01	1.00	10.00	0.40	2.00	33.3839
0.80	0.10	0.01	1.00	10.00	0.40	2.00	32.6549
0.85	0.10	0.01	1.00	10.00	0.40	2.00	31.9768
0.90	0.10	0.01	1.00	10.00	0.40	2.00	31.3433
0.95	0.10	0.01	1.00	10.00	0.40	2.00	30.7493
1.00	0.10	0.01	1.00	10.00	0.40	2.00	30.1902

Table 2: Effects of slip parameter (h) on the skin-friction

P_r	A	ε	t	n	h	Gr	C_f
0.71	0.10	0.01	1.00	10.00	0.40	2.00	34.0090
0.71	0.10	0.01	1.00	10.00	0.50	2.00	29.7330
0.71	0.10	0.01	1.00	10.00	0.60	2.00	26.4243
0.71	0.10	0.01	1.00	10.00	0.70	2.00	23.7863
0.71	0.10	0.01	1.00	10.00	0.80	2.00	21.6326
0.71	0.10	0.01	1.00	10.00	0.90	2.00	19.8404
0.71	0.10	0.01	1.00	10.00	1.00	2.00	18.3253

Table 3: Effect of thermal conductivity (α) on the skin-friction

P_r	A	ε	t	n	h	Gr	C_f
0.71	0.10	0.01	1.00	10.00	0.40	2.00	34.0090
0.71	0.20	0.01	1.00	10.00	0.40	2.00	35.0130
0.71	0.30	0.01	1.00	10.00	0.40	2.00	35.9497
0.71	0.40	0.01	1.00	10.00	0.40	2.00	36.8288
0.71	0.50	0.01	1.00	10.00	0.40	2.00	37.6580
0.71	0.60	0.01	1.00	10.00	0.40	2.00	38.4437
0.71	0.70	0.01	1.00	10.00	0.40	2.00	39.1909

Table 4: Effects of thermal Grashof number (Gr) on the skin-friction

P_r	A	ε	t	n	h	Gr	C_f
0.71	0.10	0.01	1.00	10.00	0.40	2.0	34.0090
0.71	0.10	0.01	1.00	10.00	0.40	2.5	42.5112
0.71	0.10	0.01	1.00	10.00	0.40	3.0	51.0135
0.71	0.10	0.01	1.00	10.00	0.40	3.5	59.5157
0.71	0.10	0.01	1.00	10.00	0.40	4.0	68.0180
0.71	0.10	0.01	1.00	10.00	0.40	4.5	76.5202
0.71	0.10	0.01	1.00	10.00	0.40	5.0	85.0224

In figure 1, which is the velocity profile for diverse values of the slip parameter (h), when the value of h was steadily increased, the flow speed slowly increased. This implies that at an increasing rate of the slip parameter, the fluid flow is increased as well.

Figure 2 stands for the velocity profile of an enhanced thermal buoyancy force. When this is done, it is obvious that the flow rate increased. This happens because the temperature of the fluid was raised as a result of an increasing thermal Grashof number which in turn reduces the viscosity of the fluid. Hence, the faster rate of fluid flow.

As a result of increasing thermal conductivity (α) of the fluid (according to figure 3), the fluid was able to conduct heat faster which resulted into the boost in its temperature. This reduced the fluid's viscosity as well and definitely speeded up the velocity of the fluid.

An increasing Prandtl number means that the thermal conductivity of the fluid is reduced. In any case, when this happens, the temperature of the fluid is reduced and the viscosity increased. So, the flow velocity is afterward reduced. This is shown in figure 4.

Table 1 represents the effect of the Prandtl number on the skin-friction. As in the case of the fluid's velocity, an increasing prandtl number lowers the skin-friction.

In table 2 however, which is the effect of the slip parameter on the skin-friction. As the values of h increases, the fluid was able to move more freely through the medium with little hindrance at the boundary layer, hence the skin-friction was reduced.

According to table 3 and 4, which are the effects of thermal Grashof number (Gr) and thermal conductivity (α) on the skin-friction (C_f), the two parameters caused the temperature of the fluid to increase and then in turn enhanced the skin-friction.

5.0 Conclusion

The study of an unsteady free convective slip flow of a fluid, incompressible and viscous pass a porous medium along an infinite vertical plate with a time dependent thermal conductivity was done. The suction velocity is taken to be constant and the flow channel is assumed to be free from any magnetic effect. The non-dimensional governing equations were converted to ordinary differential equations by perturbation method and then solved analytically. The effects of resulted various governing physical parameters were studied on the velocity of the fluid and skin-friction. It was observed that the fluid velocity increased with increase in the thermal Grashof number (Gr), the slip parameter (h) and thermal conductivity parameter (s), but decreased when the Prandtl number increased. Moreover, hike in the value of thermal conductivity and buoyancy force increased skin-friction while that of slip parameter and prandtl number lowers the skin-friction.

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