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An Economic Order Quantity (EOQ) Model for Items with Linear Demand, Imperfect Quality and Inspection Errors

[1]Usman T., [2]Dari S., [3]Stephen K., [4]Magaji A. S. and [5]Nasiru A.
[1,2,3,4]Department of Mathematical Sciences, Kaduna State University, Kaduna.
[5]Department of Mathematical Sciences, Nigeria Defense Academy, Kaduna.

[1]usmanmath309@gmail.com, [2]sanisdari@yahoo.com, [3]keturahstephen14@gmail.com,
[4]abu_magaji@yahoo.com, [5]Nasiru A. nabdullahi@nda.edu.ng

Abstract
In practice, when a lot size of stock is received, an inspection process is necessary to identify the defective items. In addition, the inspection process itself is not error-free and it may contain misclassification errors. In this paper, an economic order quantity model for imperfect quality items under screening errors is studied where the demand is assumed to be linearly dependent on time. It is also assumed in this paper that shortages are not allowed. The best cycle length which optimizes the total profit is obtained. For applicability of the developed model, a numerical example is illustrated and sensitivity analysis was carried out to show the effect on some of the system parameters (i.e. demand, holding cost, screening rate and selling price) on the total variable cost.

Keywords: Imperfect process Misclassification errors EOQ

Introduction

Inventory system is one of the main streams of operations research which is essential in business enterprises and industries. Interest in the subject is constantly increasing and its development in recent years closely parallels the development of operations research in general (Jacobson, 1952). Wang *et al.* (2010) claim that "more operations research has been directed towards inventory control than any other problem area in business and industry and imperfect items inventory have been gaining attention by researchers". Traditional, inventory models are developed mostly to obtained an economic (optimal) order quantity (EOQ) or economic production quantity (EPQ) based on the ordering/setup cost and the inventory carrying cost which will optimize the total variable cost per unit time. Some authors make many assumptions while coming up with a closed form solution for the most economical batch size in a stock or a production facility. One assumption is that the items produced by the facility are all of a perfect quality and the screening process that identifies the defective items in a lot is error-free (i.e., the defective items from a lot can be screened out through 100% inspection), which is an idealistic approach. In practice, the production lot may contain a substantial number of defective items, possibly because of weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit. The Raouf (1983) approach was used to suggest that an inspector could make two classifications while screening, i.e. an item may be classified as defective or non-defective. However, because the inspection process is not error-free, a good item may be classified as defective, i.e. a Type I error, while a defective item may be classified as good, i.e. a Type II error.

In practical situations, it is unreliable to assume that 100% of ordered items are perfect and the inspectors are error-free. Thus, it is necessary to consider inventory problems for imperfect items under screening-errors. Many mathematical models have been developed for controlling inventory. However, the formulation of the order inventory policies for controlling the order size has received relatively little attention. In practice, demand and inventory level may influence the order size. A situation in which the demand decreases or increases may cause the retailer to decrease or increase their order size as well. Also, the order size may either increase or decrease with the inventory level. Thus, the effect of the order size on inventory systems warrants further study. The standard EOQ model assumes a constant and known demand rate over an infinite planning horizon. However, most items experience a variable demand rate. Hence, the EOQ model must obviously be modified. Many studies have extended the EOQ model in order to accommodate time varying demand patterns. Mandal (2008) discussed the inventory model with an inventory level dependent rate. He developed a production inventory model with the assumption that the finite production rate is proportional to both demand rate and the inventory level, when the demand rate follows a time function. Raouf (1983) studied human error in inspection planning and came up with one of the first inspection plans with misclassifications for multi-characteristic critical components. He suggested repeating the cycle of inspection to ensure the product quality and determined an optimal number of inspection cycles based on the cost of inspection and misclassifications. Porteus (1986) studied the effect of defective items on the basic EOQ model. He assumed that there was a fixed probability that the process would go out-of-control. Rosenblatt (1986) assumed that the time between the in-control and the out-of-control state of a process follows an exponential distribution and that the defective items are reworked instantaneously. Raouf (1983), Porteus (1986) and Rosenblatt (1986) suggested that producing in smaller lots when the process is not perfect and assumed that the demand is constant. In a later paper, Lee (1987) studied a joint-lot sizing and inspection policy for an EOQ model with a fixed percentage of defective products. While most of the literature in this area deals with deterministic problems, many researchers have discussed stochastic production yield and demand rates. In the year 2000, Salameh (2000) has been receiving attention, he studied a joint lot sizing and inspection policy for an EOQ model when a random proportion of the units in a lot are defective. He assumed a 100% screening process with no human error and suggested that poor-quality items should be salvaged as a single batch at the end of the 100% screening process. Salameh (2000) model will be referred to as the S&J model in the rest of the paper. Duffuaa (2002) suggested an inspection plan for these critical components (i.e. which causes disaster or a high cost upon failure) where an inspector can commit a number of. Salameh (2000) and Duffuaa (2002) extended the Raouf (1983) inspection plan for the case of a number of misclassifications. This was a realistic approach where an inspector can classify an item to be non-defective, reworkable or scrap. Goyal (2002) suggested a simpler approach to the S & J model, which he used to determine an integrated vendor–buyer inventory policy for an item with imperfect quality. They suggested that the order size should be delivered in n sub lots. Chiu (2003) considered the effect of reworking defective items on the EPQ model with backlogging allowed. He suggested that some of the defective items should be reworked while the rest should be scrapped. Inderfurth (2004) determined an optimal production policy for a uniformly distributed demand and yield rate and discussed some managerial aspects of inspection. Huang (2004) presented an integrated vendor–buyer inventory model for an unreliable process of inspection. He derived an analytic solution for the optimal order quantity and the number of deliveries in each purchase order. In a later paper, Duffuaa (2005) carried out a sensitivity analysis to study the effect of different types of misclassifications on the optimal inspection plan. Papachristos (2006) discussed the non- shortages in inventory models where the proportion of defective items is a random variable. They proposed an alternative to the S & J model by speculating on the timing of withdrawing and selling the imperfect lot. Rekik (2007) extended the work of Inderfurth (2004) for two cases: (a) an additive errors case where the variability of errors is independent of the order quantity, and (b) a multiplicative errors case where the variability of errors is proportional to the order quantity. Liao (2007) studied imperfect production processes that require maintenance. They considered two states, namely the in-control and the out-of-control state of the production process. The maintenance process would either worsen the production system or bring it to a perfect state. They showed that there exists a unique optimal number of imperfect maintenance processes (N) before the production system is brought to the perfect state. Maddah (2008) corrected a flaw in the S & J model by using renewal theory. They came up with simpler expressions for the expected profit and the order quantity. The S & J model suggested that the imperfect items are not reworked but just withdrawn from the received lot. It is also assumed that there is no human error in the screening process. There may be many sources of errors in inspection, one of which is inaccuracy in records. (K¨ ok, 2007) Discussed inaccuracies in inventory records. They proved that an inspection adjusted base-stock policy is optimal for a single period problem, where inspection is performed if the inventory recorded is less than a threshold level. Atali (2009) also modeled the discrepancies between actual and the recorded inventories in retail and distribution environments. Khan et al; (2011), Jia-Tzer and Lie-Fern (2013), Skouri et al; (2014), Hadi (2018), Muhammad (2019), Hadi et al; (2019) and Muhammad et al; (2020) also contributed to the inventory scenarios with inspection of items. Khan et al., (2011) constructed an EOQ model for items with imperfect quality and inspection errors where the demand is constant. But in reality, demand rate of an item is not always constant, as it is subject to variations due to climate conditions, price, income of people, style, taste, population, etc. to mention but a few which can cause variation in the demand for an item. Thus, an EOQ model for items with imperfect quality and inspection errors with linear demand is developed in this paper. Hence this paper extends the work of Khan et al. (2011). To the authors' understanding, this type of model (with the underline assumptions above) has not yet been considered by any of the researchers/scientists in inventory literature. The model developed in this paper will determine the holding cost, the best cycle length and the expected total profit. Numerical

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example will be given to illustrate the applicability of the model and sensitivity analysis will also be carried out to see the effect of changes on some system parameters.

Model Development

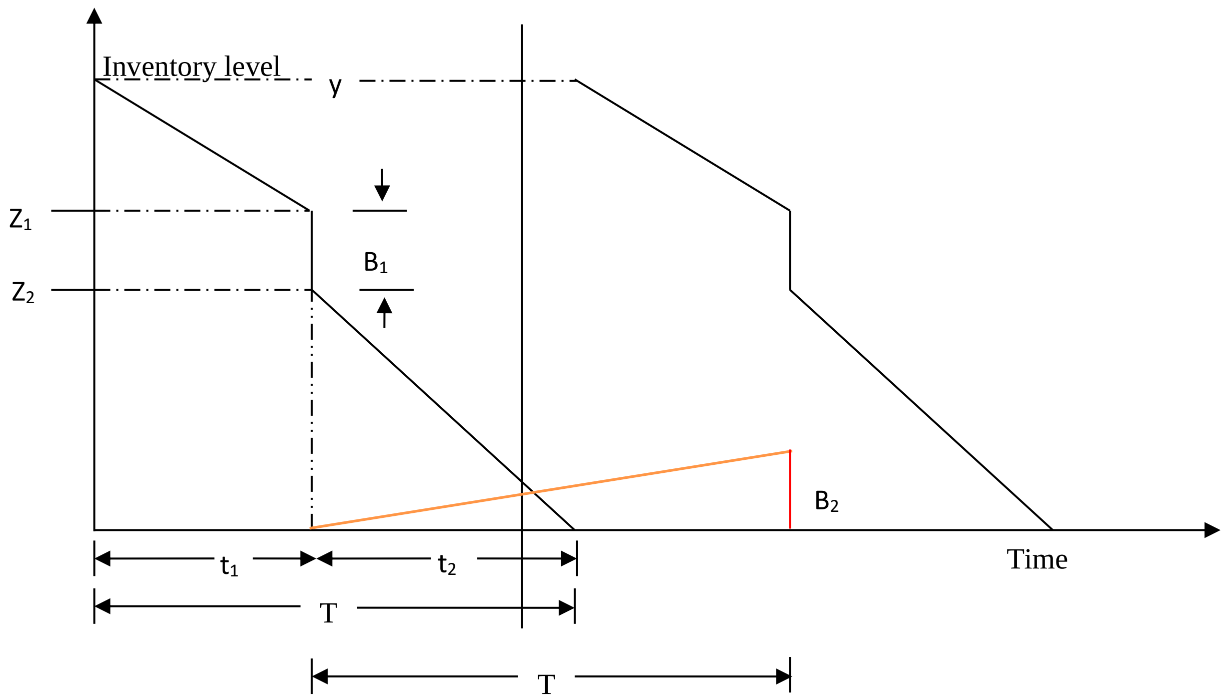

Fig 1: Behavior of inventory level over time

Consider a lot of size y being delivered to the buyer. It is assumed that each lot contains a fixed proportion p of defective items. An inspector screens out the defective items from the lot with fixed rate of misclassifications. That is, a proportion m_1 of non-defective items are classified to be defective and a proportion m_2 of defective items are classified to be non-defective .It is assumed that the probability density functions, $f\,(p)$, $f\,(m_1)$ and $f\left(m_2\right)$ are known. It is also assumed that the items that are returned from the market are stored with those that are classified as defective by the inspector. They are all sold as a single batch at the end of each cycle at a discounted price. The behavior of the inventory level is illustrated in Fig 1, where T is the cycle length, B_1 is the batch classified as defective by the inspector while B_2 is the batch of returned units from the market accumulated over T. An optimal inventory policy is determined using the total revenues and the total costs. The costs considered in the model are the procurement cost, screening cost, misclassification cost and the inventory carrying cost.

Assumptions and Notations

The following notations are used throughout:

D Number of units demanded per year (demand rate)

α	The initial demand
β	The rate of change of demand
y	Order size
c	Unit variable cost
K	Fixed ordering cost
s	unit selling price of a non-defective item
v	unit selling price of a defective item
x	screening rate
d	unit screening cost
h	unit holding cost
T	cycle length
m_1	Probability of Type I error (classifying a non-defective item as defective)
m_2	Probability of Type II error (classifying a defective item as non-defective)
p	Probability that an item is defective
t_1	Inspection time in a cycle
t_2	The remaining time in a cycle, after the defective items are screened out
$f(p)$	Probability density function of p
$f(m_1)$	Probability density function of m_1
$f(m_2)$	Probability density function of m_2
B_1	Number of items that are classified as defective in one cycle
B_2	Number of defective items that are returned from the market in one cycle
c_a	Cost of accepting a defective item
c_r	Cost of rejecting a non-defective item

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The following assumptions were made:

The screening and consumption of the inventory continues until time t_1, after which all the defectives (B_1) are withdrawn from inventory as a single batch and are sold to the secondary market.
The consumption process continues at the demand rate until the end of cycle time T.
Due to inspection error, some of the items used to fulfill the demand would be defective. These defective items are later returned to the inventory and are shown in Fig.1 as B_2.
To avoid shortages, it is also assumed that the number of non-defective items is at least equal to the adjusted demand, that is the sum of the actual demand and items that are replaced for the ones returned from the market over T.
It is also assumed that demand is a linear function give as:

$$\text{Demand} = \alpha + \beta t$$

$$\text{where } \alpha > 0,\ \beta < 0,\ 0 \le t \le T$$

$$\text{Demand rate (D)} = \frac{1}{T}\int_0^T (\alpha + \beta t)dt$$

$$= \frac{1}{T}\left[\alpha t + \frac{\beta t^2}{2}\right]_0^T$$

$$= \frac{1}{T}\left[\alpha T + \frac{\beta T^2}{2}\right]$$

$$\Rightarrow D = \alpha + \frac{\beta T}{2} \quad (1)$$

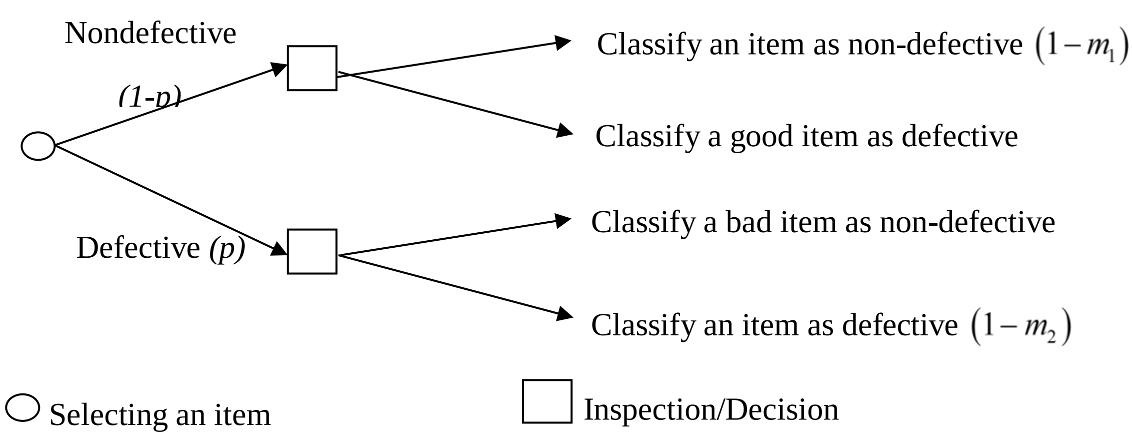

Fig: 2 Four possibilities in the inspection process

Consider now the different cases of misclassifications that an inspection process can have. There are four possibilities in such an inspection process. Those are:

Case (1): A non-defective item is classified as non-defective

Case (2): A non-defective item is classified as defective

Case (3): A defective item is classified as non-defective and

Case (4): A defective item is classified as defective.

This scenario is depicted in Fig. 2

The number of items going into different categories following these cases is given by:

Case (1): $y(1-p)(1-m_1)$

Case (2): $y(1-p)m_1$

Case (3): ypm_2

Case (4): $yp(1-m_2)$

Thus

$$y - y(1-p)m_1 - yp(1-m_2) \geq DT + pm_2 y$$

substituting $D = \alpha + \frac{\beta T}{2}$, we have

$$y - y(1-p)m_1 - yp(1-m_2) - pm_2 y \geq \alpha T + \frac{\beta T^2}{2}$$

$$\Rightarrow \qquad 2y(1-p)(1-m_1) \geq 2\alpha T + \beta T^2$$

So, for the limiting case, the cycle length can be written as

$$T = \frac{-\alpha \pm \sqrt{\alpha^2 + 2\beta y(1-p)(1-m_1)}}{\beta}$$

since T cannot be negative

$$T = \frac{-\alpha + \sqrt{\alpha^2 + 2\beta y(1-p)(1-m_1)}}{\beta} \qquad (2)$$

Now B_1 and B_2 are given by

$$\left.\begin{aligned} B_1 &= y(1-p)m_1 + yp(1-m_2) \\ B_2 &= ypm_2 \end{aligned}\right\} \qquad (2*)$$

The items in B_2 are returned from the market at the rate ypm_2 / T and are taken from the inventory with batch B_1. Therefore, the revenue from salvaging $(B = B_1 + B_2)$ items is given by

$$R_1 = vB$$
$$= v(B_1 + B_2)$$
$$= vy(1-p)m_1 + vyp(1-m_2) + vypm_2$$
$$\therefore \quad R_1 = vy(1-p)m_1 + vyp$$

The revenue from selling the good items is computed as

$$R_2 = sy(1-p)(1-m_1) + sypm_2$$

So, the total revenue is given as

$$R = R_1 + R_2$$
$$= vy\left(1-p\right)m_1 + vyp + sy\left(1-p\right)\left(1-m_1\right) + sypm_2$$
$$\therefore \quad R = y\left(1-p\right)\left\{vm_1 + s\left(1-m_1\right)\right\} + yp\left(v + sm_2\right) \tag{3}$$

Consider now the different costs of the inventory system, the procurement cost per cycle is

$$PC = K + py \tag{4}$$

Where C is the variable cost. The screening cost per cycle is the sum of the cost of inspection and misclassifications which is given by

$$I_C = dy + c_r(1-p)ym_1 + c_a pym_2 \tag{5}$$

Therefore, the total cost per cycle is given by summing up equations (4, 5 and 6)

Total cost C = $Pc + Ic + Hc$

$$C = k + cy + dy + c_r y(1-p)m_1 + c_a ypm_2$$
$$+\frac{hy}{2}\left\{\begin{array}{l}\dfrac{2\alpha+\beta T}{2x}\left(1+(1-p)m_1 + p(1-m_2)\right) \\ +T\left(1-(1-p)m_1 - p(1-m_2) - \dfrac{\left(2\alpha+\beta T\right)^2}{4xy}\right) + pm_2 T\end{array}\right\} \tag{7}$$

From Fig.1, the behaviour of different types of inventory in the order cycle.The red triangle at the bottom represents the defective lot that is returned by the market and is accumulated into the salvage lot the total profit per cycle can be written as the difference between the total revenue and total cost per cycle, that is

Total profit $\left(TP(y)\right) = R - C$

The holding cost per cycle is the cost of carrying the (i) nondefective lot, (ii) defective lot and (iii) returned lot. So, from Fig. 1 the holding cost for a cycle can be written as

$$HC = h\left\{\frac{(y-Z_1)t_1}{2} + Z_1t_1 + \frac{Z_2t_2}{2}\right\} + h\left(\frac{B_2T}{2}\right)$$

$$HC = \frac{h}{2}\left(yt_1 + Z_1t_1 + Z_2t_2\right) + \frac{h}{2}\left(B_2T\right) \qquad (*)$$

Substituting $Z_2 = Z_1 - B_1$, $t_2 = T - t_1$, $B_1 = y(1-p)m_1 + yp(1-m_2)$, $B_2 = ypm_2$, $t_1 = \frac{D}{x} = \frac{2\alpha + \beta T}{2x}$ and $Z_1 = y - Dt_1 = y - \frac{\left(2\alpha + \beta T\right)^2}{4x}$ equation $(*)$ becomes

$$HC = \frac{hy}{2}\left\{\begin{array}{l}\frac{2\alpha + \beta T}{2x}\left(1 + (1-p)m_1 + p(1-m_2)\right) \\ +T\left(1 - (1-p)m_1 - p(1-m_2) - \frac{\left(2\alpha + \beta T\right)^2}{4xy}\right)\end{array}\right\} + \frac{h}{2}\left(ypm_2T\right) \qquad (6)$$

$$TP(y) = vy(1-p)m_1 + vyp + sy(1-p)(1-m_1) + sypm_2 - \left[K + cy + dy + c_r y(1-p)m_1 + c_a ypm_2\right]$$
$$-\frac{hy}{2}\left\{\begin{array}{l}\frac{2\alpha + \beta T}{2x}\left(1 + (1-p)m_1 + p(1-m_2)\right) \\ +T\left(1 - (1-p)m_1 - p(1-m_2) - \frac{\left(2\alpha + \beta T\right)^2}{4xy}\right) + pm_2T\end{array}\right\} \qquad (8)$$

Since p, m_1 and m_2 are random variables with probability density functions $f(p)$, $f(m_1)$ and $f(m_2)$ the expected total profit can be written as

$$E\left[TP(y)\right] = vy\left(1 - E\left[p\right]\right)E\left[m_1\right] + vyE\left[p\right] + sy\left(1 - E\left[p\right]\right)\left(1 - E\left[m_1\right]\right) + syE\left[p\right]E\left[m_2\right]$$
$$-\left[K + cy + dy + c_r y\left(1 - E\left[p\right]\right)E\left[m_1\right] + c_a yE\left[p\right]E\left[m_2\right]\right]$$
$$-\frac{hy}{2}\left\{\begin{array}{l}\frac{2\alpha + \beta E[T]}{2x}\left(1 + \left(1 - E\left[p\right]\right)E\left[m_1\right] + E\left[p\right]\left(1 - E\left[m_2\right]\right)\right) \\ +E[T]\left(1 - \left(1 - E\left[p\right]\right)E[m_1] - E\left[p\right]\left(1 - E\left[m_2\right]\right) - \frac{\left(2\alpha + \beta E\left[T\right]\right)^2}{4xy}\right) \\ +E\left[p\right]E\left[m_2\right]E\left[T\right]\end{array}\right\} \qquad (9)$$

from equation (2), the expected cycle length would be

$$E[T]=\left(-\alpha+\left(\alpha^2+2\beta y\left(1-E[p]\right)\left(1-E[m_1]\right)\right)^{1/2}\right)\beta^{-1} \tag{10}$$

Using Maddah and Jaber (2008) approach, the expected annual profit for this model is

$$E\left[TPU(y)\right]=\frac{E[TP(y)]}{E[T]}$$

$$\frac{E[TP(y)]}{E[T]}=\left(\frac{vy\left(1-E[p]\right)E[m_1]+vyE[p]+sy\left(1-E[p]\right)\left(1-E[m_1]\right)+syE[p]E[m_2]-\left[K+cy+dy+c_r y\left(1-E[p]\right)E[m_1]+c_a yE[p]E[m_2]\right]}{E[T]}\right)$$
$$-\frac{hy}{2}\left\{\begin{array}{l}\dfrac{2\alpha+\beta E[T]}{2xE[T]}\left(1+\left(1-E[p]\right)E[m_1]+E[p]\left(1-E[m_2]\right)\right)\\ +\dfrac{E[T]}{E[T]}\left(1-\left(1-E[p]\right)E[m_1]-E[p]\left(1-E[m_2]\right)-\dfrac{\left(2\alpha+\beta E[T]\right)^2}{4xy}\right)+\dfrac{E[p]E[m_2]E[T]}{E[T]}\end{array}\right\}$$

$$=\left(\frac{vy\left(1-E[p]\right)E[m_1]+vyE[p]+sy\left(1-E[p]\right)\left(1-E[m_1]\right)+syE[p]E[m_2]-\left[K+cy+dy+c_r y\left(1-E[p]\right)E[m_1]+c_a yE[p]E[m_2]\right]}{E[T]}\right)$$
$$-\frac{hy}{2}\left\{\begin{array}{l}\dfrac{\alpha}{xE[T]}\left(1+\left(1-E[p]\right)E[m_1]+E[p]\left(1-E[m_2]\right)\right)+\dfrac{\beta}{2x}\left(1+\left(1-E[p]\right)E[m_1]+E[p]\left(1-E[m_2]\right)\right)\\ +\left(1-\left(1-E[p]\right)E[m_1]-E[p]\left(1-E[m_2]\right)-\dfrac{\left(2\alpha+\beta E[T]\right)^2}{4xy}\right)+E[p]E[m_2]\end{array}\right\}$$

$$=\left(\frac{vy\left(1-E[p]\right)E[m_1]+vyE[p]+sy\left(1-E[p]\right)\left(1-E[m_1]\right)+syE[p]E[m_2]-\left[K+cy+dy+c_r y\left(1-E[p]\right)E[m_1]+c_a yE[p]E[m_2]\right]}{\left(-\alpha+\left(\alpha^2+2\beta y\left(1-E[p]\right)\left(1-E[m_1]\right)\right)^{1/2}\right)\beta^{-1}}\right)$$
$$-\frac{hy}{2}\left\{\begin{array}{l}\dfrac{\alpha\left(1+\left(1-E[p]\right)E[m_1]+E[p]\left(1-E[m_2]\right)\right)}{x\left(-\alpha+\left(\alpha^2+2\beta y\left(1-E[p]\right)\left(1-E[m_1]\right)\right)^{1/2}\right)}+\dfrac{\beta}{2x}\left(1+\left(1-E[p]\right)E[m_1]+E[p]\left(1-E[m_2]\right)\right)\\ +\left(1-\left(1-E[p]\right)E[m_1]-E[p]\left(1-E[m_2]\right)-\dfrac{\left(\alpha+\left(\alpha^2+2\beta y\left(1-E[p]\right)\left(1-E[m_1]\right)\right)^{1/2}\right)^2}{4xy}\right)+E[p]E[m_2]\end{array}\right\}$$

$$E[TPU(y)]=\left(\frac{y\{v(1-E[p])E[m_1]+vE[p]+s(1-E[p])(1-E[m_1])+sE[p]E[m_2]\}-[c+d+c_r(1-E[p])E[m_1]+c_aE[p]E[m_2]]-K}{\left(-\alpha+\left(\alpha^2+2\beta y(1-E[p])(1-E[m_1])\right)^{1/2}\right)\beta^{-1}}\right)$$

$$-\frac{hy}{2}\left\{\begin{array}{l}\dfrac{\alpha\left(1+(1-E[p])E[m_1]+E[p](1-E[m_2])\right)}{x\left(-\alpha+\left(\alpha^2+2\beta y(1-E[p])(1-E[m_1])\right)^{1/2}\right)}+\dfrac{\beta}{2x}\left(1+(1-E[p])E[m_1]+E[p](1-E[m_2])\right)\\ +\left(1-(1-E[p])E[m_1]-E[p](1-E[m_2])-\dfrac{\left(\alpha+\left(\alpha^2+2\beta y(1-E[p])(1-E[m_1])\right)^{1/2}\right)^2}{4xy}\right)+E[p]E[m_2]\end{array}\right\} \quad (**)$$

Let

$M=v(1-E[p])E[m_1]+vE[p]+s(1-E[p])(1-E[m_1])+sE[p]E[m_2]-c-d-c_r(1-E[p])E[m_1]-c_aE[p]E[m_2]$,

$N=(1-E[p])E[m_1]+E[p](1-E[m_2])$, $Q=E[p]E[m_2]$ and $L=(1-E[p])(1-E[m_1])$

Equation $(**)$ becomes

$$ETPU(y)=\frac{(yM-K)}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\beta^{-1}}-\frac{hy}{2}\left\{\frac{(1+N)\left[\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right]}{2x\beta^{-1}\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)}+1-N-\frac{\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy}+Q\right\} \quad (11)$$

Optimality Condition

The optimal order size that represents the maximum annual profit, is determined by setting the first derivative equal to zero (which is the necessary condition for optimality) and solving for *y*.

The first derivative of 11 is given by:

$$ETPU'(y)=\frac{M\beta\left(\alpha^2+2\beta yL\right)^{1/2}\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)-(yM-K)\beta^2L}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}-\frac{h}{2}\left\{\frac{(1+N)\left[\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right]}{2x\beta^{-1}\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)}+1-N-\frac{\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy}+Q\right\}$$

$$+\frac{hy}{2}\left\{-\frac{(1+N)\alpha\beta^2L}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}+\frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}}\right\}$$

$$ETPU'(y)=\frac{M\beta}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)}-\frac{(yM-K)\beta^2L}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}-\frac{h}{2}\left\{\frac{(1+N)\left[\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right]}{2x\beta^{-1}\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)}+1-N-\frac{\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy}+Q\right\}$$
$$+\frac{hy}{2}\left\{-\frac{(1+N)\alpha\beta^2L}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}+\frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}}\right\} \qquad (12)$$

Equating (12) to zero, we get

$$\frac{M\beta}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)}-\frac{(yM-K)\beta^2L}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}-\frac{h}{2}\left\{\frac{(1+N)\left[\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right]}{2x\beta^{-1}\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)}+1-N-\frac{\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy}+Q\right\}$$
$$+\frac{hy}{2}\left\{-\frac{(1+N)\alpha\beta^2L}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}+\frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}}\right\}=0 \qquad (13)$$

If other parameters are given, equation (13) can be used to find the best y which optimizes the expected annual profit provided that the second derivative is less than zero (sufficient condition for optimality). The second derivative is given by

$$ETPU''(y)=-\frac{M\beta^2L}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}-\frac{\beta^2L\left[M\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left(\alpha^2+2\beta yL\right)-\beta L(yM-K)\left\{3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right\}\right]}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{\frac{3}{2}}}$$
$$-\frac{h}{2}\left\{-\frac{\alpha\beta^2L(1+N)}{x\left(\alpha^2+2\beta yL\right)^{1/2}\left(-\alpha+\left(\alpha^2-2\beta yL\right)^{1/2}\right)^2}+\frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}}\right\}+\frac{h}{2}\left\{-\frac{(1+N)\alpha\beta^2L}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}+\frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}}\right\}$$
$$+\frac{hy}{2}\left\{+\frac{(1+N)\alpha\beta^3L^2\left[3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right]}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{3/2}}+\frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left[-3\beta yL\left(\alpha^2+2\beta yL\right)^{1/2}-2\alpha^2\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)-5\beta yL\right]}{4xy^3\left(\alpha^2+2\beta yL\right)^{3/2}}\right\}$$

$$= -\frac{M\beta^2 L}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}} - \frac{\beta^2 L\left[M\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left(\alpha^2+2\beta yL\right)-\beta L\left(yM-K\right)\left\{3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right\}\right]}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{\frac{3}{2}}}$$

$$-\frac{h}{2}\left\{\begin{array}{l} -\dfrac{\alpha\beta^2 L(1+N)}{x\left(\alpha^2+2\beta yL\right)^{1/2}\left(-\alpha+\left(\alpha^2-2\beta yL\right)^{1/2}\right)^2} + \dfrac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}} + \dfrac{(1+N)\alpha\beta^2 L}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}} - \dfrac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}} \\ -\dfrac{(1+N)\alpha\beta^3 L^2 y\left[3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right]}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{3/2}} - \dfrac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left[-3\beta yL\left(\alpha^2+2\beta yL\right)^{1/2}-2\alpha^2\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)-5\beta yL\right]}{4xy^2\left(\alpha^2+2\beta yL\right)^{3/2}} \end{array}\right\}$$

$$= -\frac{M\beta^2 L\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left(\alpha^2+2\beta yL\right)+\beta^2 L\left[M\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left(\alpha^2+2\beta yL\right)-\beta L\left(yM-K\right)\left\{3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right\}\right]}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}$$

$$-\frac{h}{2}\left\{\begin{array}{l} -\dfrac{\alpha\beta^2 L(1+N)}{x\left(\alpha^2+2\beta yL\right)^{1/2}\left(-\alpha+\left(\alpha^2-2\beta yL\right)^{1/2}\right)^2} + \dfrac{(1+N)\alpha\beta^2 L}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}} + \dfrac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}} - \dfrac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2}{4xy^2\left(\alpha^2+2\beta yL\right)^{1/2}} \\ -\dfrac{(1+N)\alpha\beta^3 L^2 y\left[3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right]}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{3/2}} - \dfrac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left[-3\beta yL\left(\alpha^2+2\beta yL\right)^{1/2}-2\alpha^2\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)-5\beta yL\right]}{4xy^2\left(\alpha^2+2\beta yL\right)^{3/2}} \end{array}\right\}$$

$$= -\frac{2M\beta^2 L\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left(\alpha^2+2\beta yL\right)-\beta^3 L^2\left(yM-K\right)\left\{3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right\}}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}$$

$$-\frac{h}{2}\left\{-\frac{(1+N)\alpha\beta^3 L^2 y\left[3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right]}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{3/2}} - \frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left[-3\beta yL\left(\alpha^2+2\beta yL\right)^{1/2}-2\alpha^2\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)-5\beta yL\right]}{4xy^2\left(\alpha^2+2\beta yL\right)^{3/2}}\right\}$$

$$= -\frac{2M\alpha^2\beta^2 L\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)-4M\alpha\beta^3L^2y+4M\beta^3L^2y\left(\alpha^2+2\beta yL\right)^{1/2}-3M\beta^3L^2y\left(\alpha^2+2\beta yL\right)^{1/2}+M\alpha\beta^3L^2y+K\beta^3L^2\left\{3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right\}}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}$$

$$-\frac{h}{2}\left\{-\frac{(1+N)\alpha\beta^3 L^2 y\left[3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right]}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{3/2}} + \frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left[3\beta yL\left(\alpha^2+2\beta yL\right)^{1/2}+2\alpha^2\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)+5\beta yL\right]}{4xy^2\left(\alpha^2+2\beta yL\right)^{3/2}}\right\}$$

$$\therefore \qquad ETPU''(y) = -\frac{2M\alpha^2\beta^2 L\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)+3M\beta^3L^2y\left(-3\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)+K\beta^3L^2\left\{3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right\}}{\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^2\left(\alpha^2+2\beta yL\right)^{1/2}}$$

$$-\frac{h}{2}\left\{\frac{\alpha\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)\left[3\beta yL\left(\alpha^2+2\beta yL\right)^{1/2}+2\alpha^2\left(\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)+5\beta yL\right]}{4xy^2\left(\alpha^2+2\beta yL\right)^{3/2}} - \frac{(1+N)\alpha\beta^3 L^2 y\left[3\left(\alpha^2+2\beta yL\right)^{1/2}-\alpha\right]}{x\left(-\alpha+\left(\alpha^2+2\beta yL\right)^{1/2}\right)^3\left(\alpha^2+2\beta yL\right)^{3/2}}\right\} < 0 \qquad (13)$$

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Since $K>0$, $\alpha>0$, $\beta<0$, $0<E[p]<1$, $0<E[m_1]<1$, and $0<E[m_2]<1$ then ***ETPU"(y)<***0 for every $y>0$, suggesting that the annual profit in (11) is concave.

Numerical Analysis

Consider a production system that replenishes the buyer's orders instantly. This system is not perfect, i.e. it produces some defective items. The inspection process that screens out the defective items is also imperfect. The probability density functions for the fraction of defective items and the inspection errors are mostly taken from the history of a supplier/machine and workers. In the numerical example, the following input parameters are assumed

α	10units/year
β	3
c	₦5/unit
K	₦300/cycle
s	₦150/unit
v	₦50/unit
x	100 unit/min
d	₦2/unit
h	₦5/unit
c_a	₦15/unit
c_r	₦10/unit

$$f(p) \quad \begin{cases} 25, & 0 \le p \le 0.5 \\ 0, & \text{otherwise} \end{cases} \quad \Rightarrow \quad E(p) = 0.02$$

$$f(m_1) \quad \begin{cases} 25, & 0 \le p \le 0.5 \\ 0, & \text{otherwise} \end{cases} \quad \Rightarrow \quad E(m_1) = 0.02$$

$$f(m_2) \quad \begin{cases} 25, & 0 \le p \le 0.5 \\ 0, & \text{otherwise} \end{cases} \quad \Rightarrow \quad E(m_2) = 0.02$$

Substituting above values into $t_1, t_2, (2), (2*), (11)$ and (13) respectively, we obtained the inspection time in a cycle (t_1), the remaining time in a cycle after the defective items are screened out (t_2), the total inventory cycle length T, the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the annual expected profit ($ETPU(y)$) and the optimal values of the order size (y) as:

t_1	0.48 days
t_2	25.10 days
T	25.58 days
B_1	50.52 unit
B_2	0.51 unit
$ETPU(y)$	₦3883.15/year
y	1289 units

Sensitivity Analysis

Sensitivity analysis was carried out the effect of changes in some system parameters (α, β, h, c, d, s and v) on; the inspection time in a cycle (t_1), the remaining time in a cycle after the defective items are screened out (t_2), the total inventory cycle length T, the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the annual expected profit ($ETPU(y)$) and the optimal values of the order size (y) as follows, using the numerical example above.

Table 1: Sensitivity Analysis

Parameter	% change in parameter	% change in y	% change in T	% change in t_1	% change in t_2	% change in B_1	% change in B_2	% change in $E[TPU]$
α	10	-0.23274	-1.27485	1.055573	-1.31977	-0.23274	-0.23274	2.081177
	5	-0.23274	-0.7042	0.47477	-0.72691	-0.23274	-0.23274	1.037898
	2	0	-0.23022	0.230668	-0.23909	0	0	0.414525
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	-2	0	0.230811	-0.23046	0.239686	0	0	-0.41368
	-5	0.232739	0.70793	-0.47188	0.730689	0.232739	0.232739	-1.0326
	-10	0.232739	1.28986	-1.04379	1.334824	0.232739	0.232739	-2.05983
β	10	10.2405	1.175913	8.95904	1.025905	10.2405	10.2405	7.955697
	5	5.120248	0.614407	4.478176	0.53996	5.120248	5.120248	3.975866
	2	2.094647	0.278024	1.811444	0.248489	2.094647	2.094647	1.589825
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**

	-2	-2.09465	-0.28876	-1.81124	-0.2594	-2.09465	-2.09465	-1.58909
	-5	-5.12025	-0.67512	-4.47549	-0.60186	-5.12025	-5.12025	-3.9712
	-10	-10.2405	-1.42038	-8.94726	-1.27531	-10.2405	-10.2405	-7.93687
h	10	-17.4554	-10.1933	-8.08639	-10.2339	-17.4554	-17.4554	-7.25731
	5	-9.30954	-5.31629	-4.21754	-5.33746	-9.30954	-9.30954	-3.80308
	2	-3.95656	-2.22825	-1.76783	-2.23712	-3.95656	-3.95656	-1.56638
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	-2	4.189294	2.312415	1.834386	2.321651	4.189294	4.189294	1.630868
	-5	10.93871	5.943471	4.714838	5.967171	10.93871	10.93871	4.207016
	-10	23.73933	12.54189	9.949505	12.59185	23.73933	23.73933	8.885341
c	10	-0.69822	-0.39002	-0.30942	-0.39158	-0.69822	-0.69822	-0.64764
	5	-0.46548	-0.25986	-0.20628	-0.26088	-0.46548	-0.46548	-0.32411
	2	-0.23274	-0.12986	-0.10314	-0.13038	-0.23274	-0.23274	-0.12971
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	-2	0.232739	0.129707	0.102726	0.13022	0.232739	0.232739	0.129766
	-5	0.465477	0.259266	0.205658	0.26032	0.465477	0.465477	0.324583
	-10	0.698216	0.388677	0.308177	0.390222	0.698216	0.698216	0.649717

parameter	% change in parameter	% change in y	% change in T	% change in t_1	% change in t_2	% change in B_1	% change in B_2	% change in $E[TPU]$
d	10	-0.2327	-0.1298	-0.1031	-0.1303	-0.2327	-0.2327	-0.2593
	5	-0.2327	-0.1298	-0.1031	-0.1303	-0.2327	-0.2327	-0.1297
	2	0	0	0	0	0	0	-0.0519
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	-2	0	0	0	0	0	0	0.0519
	-5	0.2327	0.1297	0.1027	0.1302	0.2327	0.2327	0.1298
	-10	0.2327	0.1297	0.1027	0.1302	0.2327	0.2327	0.2596
s	10	22.110	11.721	9.2986	11.77	22.110	22.110	19.562
	5	10.706	5.8201	4.6171	5.843	10.706	10.706	9.5648
	2	4.1893	2.3124	1.8344	2.322	4.189294	4.189294	3.774069
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	-2	-4.18929	-2.36072	-1.87283	-2.37013	-4.18929	-4.18929	-3.70496
	-5	-10.2405	-5.86243	-4.65076	-5.88579	-10.2405	-10.2405	-9.13286
	-10	-19.7828	-11.6298	-9.22609	-11.6761	-19.7828	-19.7828	-17.8341

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	10	0.232739	0.129707	0.102726	0.13022	0.232739	0.232739	0.257032
	5	0.232739	0.129707	0.102726	0.13022	0.232739	0.232739	0.128467
	2	0	0	0	0	0	0	0.051373
v	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	-2	0	0	0	0	0	0	-0.05137
	-5	-0.23274	-0.12986	-0.10314	-0.13038	-0.23274	-0.23274	-0.12842
	-10	-0.23274	-0.12986	-0.10314	-0.13038	-0.23274	-0.23274	-0.25672

From Table 1 above, it can be observed that:

As the initial demand (α) increases, the optimal value of order size (y), the total inventory cycle length T, the remaining time in a cycle after the defective items are screened out (t_2), the number of items that are classified as defective in one cycle (B_1) and the number of defective items that are returned from the market in one cycle (B_2) all increase, while the inspection time in a cycle (t_1) and the annual expected profit ($ETPU(y)$) decrease. As the initial demand rate (β) increases, the optimal value of order size (y), the total inventory cycle length T, the remaining time in a cycle after the defective items are screened out (t_2), the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the inspection time in a cycle (t_1) and the annual expected profit ($ETPU(y)$) all increase.
As the unit holding cost (h) increases, the optimal value of order size (y), the total inventory cycle length T, the remaining time in a cycle after the defective items are screened out (t_2), the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the inspection time in a cycle (t_1) and the annual expected profit ($ETPU(y)$) all decrease. As the unit screening rate (d) increases, the optimal value of order size (y), the total inventory cycle length T, the remaining time in a cycle after the defective items are screened out (t_2), the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the inspection time in a cycle (t_1) and the annual expected profit ($ETPU(y)$) all decrease. As the unit selling price of a non-defective item (s) increases, the optimal value of order size (y), the total inventory cycle length T, the remaining time in a cycle after the defective items are screened out (t_2), the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the inspection time in a cycle (t_1) and the annual expected profit ($ETPU(y)$) all increase. As the unit selling price of a defective item (v) increases, the optimal value of order size (y), the total inventory cycle length T, the remaining time in a cycle after the defective items are screened out (t_2), the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the inspection time in a cycle (t_1) and the annual expected profit ($ETPU(y)$) all increase.
In general, the optimal value of order size (y), the total inventory cycle length T, the remaining time in a cycle after the defective items are screened out (t_2), the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the

market in one cycle (B_2), the inspection time in a cycle (t_1) and the annual expected profit ($ETPU(y)$) all are directly proportional to the initial demand rate (β), the unit selling price of a non-defective item (s) and the unit selling price of a defective item (v) while they are inversely proportional to the unit holding cost (h) and the unit screening rate (d).

Conclusion

In this paper, an economic order quantity model for imperfect quality items with linear demand under screening errors is studied. For authenticity of the developed model, a numerical example is illustrated and the sensitivity analysis is also carried out to show the effect on some of the system parameters on the total variable cost. From the sensitivity analysis, it is observed that the inspection time in a cycle (t_1), the remaining time in a cycle after the defective items are screened out (t_2), the total inventory cycle length T, the number of items that are classified as defective in one cycle (B_1), the number of defective items that are returned from the market in one cycle (B_2), the annual expected profit ($ETPU(y)$) and the optimal values of the order size (y) are more sensitive to changes in the unit holding cost (h) and the unit selling price of a non-defective item (s) while they are less sensitive to changes in the unit selling price of a defective item (v), the unit screening rate (d) and the initial demand (α).

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